Topology Control in Heterogeneous Wireless Networks: Problems and Solutions

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Abstract—Previous work on topology control usually assumes homogeneous wireless nodes with uniform transmission ranges. In this paper, we propose two localized topology control algorithms for heterogeneous wireless multi-hop networks with non-uniform transmission ranges: Directed Relative Neighborhood Graph (DRNG) and Directed Local Minimum Spanning Tree (DLMST). In both algorithms, each node selects a set of neighbors based on the locally collected information. We prove that (1) the topologies derived under DRNG and DLMST preserve the network connectivity; (2) the out degree of any node in the resulting topology by DLMST is bounded; while the out degree of nodes in the topology by DRNG is not bounded; and (3) the topologies generated by DRNG and DLMST preserve the network bi-directionality.

I. INTRODUCTION

Energy efficiency [1] and network capacity are perhaps two of the most important issues in wireless ad hoc networks and sensor networks. Topology control algorithms have been proposed to maintain network connectivity while reducing energy consumption and improving network capacity. The key idea to topology control is that, instead of transmitting using the maximal power, nodes in a wireless multi-hop network collaboratively determine their transmission power and define the network topology by forming the proper neighbor relation under certain criteria.

By enabling wireless nodes to use adequate transmission power (which is usually much smaller than the maximal transmission power), topology control can not only save energy and prolong network lifetime, but also improve spatial reuse (and hence the network capacity) [2] and mitigate the MAC-level medium contention [3]. Several topology control algorithms [3]–[10] have been proposed to create power-efficient network topology in wireless multi-hop networks with limited mobility (a summary is given in Section III). However, most of them assume homogeneous wireless nodes with uniform transmission ranges (except [4]).

The assumption of homogeneous nodes does not always hold in practice since even devices of the same type may have slightly different maximal transmission power. There also exist heterogeneous wireless networks in which devices have dramatically different capabilities, for instance, the communication network in the Future Combat System which involves wireless devices on soldiers, vehicles and UAVs. As will be exemplified in Section III, most existing algorithms cannot be directly applied to heterogeneous wireless multi-hop networks in which the transmission range of each node may be different.

To the best of our knowledge, this paper is the first effort to address the connectivity and bi-directionality issue in the heterogeneous wireless networks.

In this paper, we propose two localized topology control algorithms for heterogeneous wireless multi-hop networks with non-uniform transmission ranges: Directed Relative Neighborhood Graph (DRNG) and Directed Local Minimum Spanning Tree (DLMST). In both algorithms, the topology is constructed by having each node build its neighbor set and adjust its transmission power based on the locally collected information. We are able to prove that (1) the topology derived under both DRNG and DLMST preserves network connectivity, i.e., if the original topology generated by having every node use its maximal transmission power is strongly connected, then the topologies generated by both DRNG and DLMST are also strongly connected; (2) the out degree of any node in the topology by DLMST is bounded, while the out degree of nodes in the topology by DRNG may be unbounded; and (3) the topology generated by DRNG and DLMST preserves network bi-directionality, i.e., if the original topology by having every node use its maximal transmission power is bi-directional, then the topology generated by either DRNG or DLMST is also bi-directional after some simple operations.

Simulation results indicate that compared with the other known topology control algorithms that can be applied to heterogeneous networks, DRNG and DLMST have smaller average node degree (both logical and physical) and smaller average link length. The former reduces the MAC-level contention, while the latter implies a small transmission power needed to maintain connectivity.

The rest of the paper is organized as follows. In Section II, we give the network model. In Section III, we summarize previous work on topology control, and give examples to show why existing algorithms cannot be directly applied to heterogeneous networks. Following that, we present both the DRNG and DLMST algorithms in Section IV, and prove several of their useful properties in Section V. Finally, we evaluate the performance of the proposed algorithms in Section VI, and conclude the paper in Section VII.
II. NETWORK MODEL

Consider a set of nodes, \( V = \{v_1, v_2, \ldots, v_n\} \), which are randomly distributed in the 2-D plane. Let \( r_{v_i} \) be the maximal transmission range of \( v_i \). In a heterogeneous network, the maximal transmission ranges of all nodes may not be the same. Let \( r_{m_{\text{min}}} = \min_{v \in V} \{r_v\} \) and \( r_{m_{\text{max}}} = \max_{v \in V} \{r_v\} \). We denote the network topology generated by having each node use its own maximal transmission power as a simple directed graph \( G = (V(G), E(G)) \), where \( E(G) = \{(u, v) : d(u, v) \leq r_u, u, v \in V(G)\} \) is the edge set of \( G \) and \( d(u, v) \) is the Euclidean distance between node \( u \) and node \( v \). Note that \( (u, v) \) is an ordered pair representing an edge from node \( u \) to node \( v \). A unique \( id \) (such as an IP/MAC address) is assigned to each node. Here we let \( id(v_i) = i \) for simplicity.

We assume that the wireless channel is symmetric \(^1\) and obstacle-free, and each node is equipped with the capability to gather its location information via, for example, GPS for outdoor applications and pseudolite [11] for indoor applications, and other lightweight localization techniques for wireless networks (see [12] for a summary).

Before delving into the technical discussion and algorithm description, we give the definition of several terms that will be used throughout the paper.

Definition 1 (Reachable Neighborhood): The reachable neighborhood \( N_u^R \) is the set of nodes that node \( u \) can reach using its maximal transmission power, i.e., \( N_u^R = \{v \in V(G) : d(u, v) \leq r_u\} \). For each node \( u \in V(G) \), let \( G_u^R = (V(G_u^R), E(G_u^R)) \) be an induced subgraph of \( G \) such that \( V(G_u^R) = N_u^R \).

Definition 2 (Weight Function): Given two edges \((u_1, v_1), (u_2, v_2) \in E\) and the Euclidean distance function \( d(\cdot, \cdot) \), weight function \( w : E \mapsto R \) satisfies:

\[
\begin{align*}
  w(u_1, v_1) &> w(u_2, v_2) \\
  \iff d(u_1, v_1) &> d(u_2, v_2) \\
  \quad \text{or} (d(u_1, v_1) = d(u_2, v_2) &\land \max\{id(u_1), id(v_1)\} > \max\{id(u_2), id(v_2)\}) \\
  \quad \text{or} (d(u_1, v_1) = d(u_2, v_2) &\land \max\{id(u_1), id(v_1)\} = \max\{id(u_2), id(v_2)\} \\
  \quad &\land \min\{id(u_1), id(v_1)\} > \min\{id(u_2), id(v_2)\}).
\end{align*}
\]

This weight function ensures that two edges with different end nodes have different weights. Note that \( w(u, v) = w(v, u) \).

Definition 3 (Neighbor Set): Node \( v \) is a neighbor of node \( u \) under an algorithm \( A \), denoted \( u \rightarrow A v \), if and only if there exists an edge \((u, v)\) in the topology generated by the algorithm. In particular, we use \( u \rightarrow v \) to denote the neighbor relation in \( G \), \( u \rightarrow A v \) if and only if \( u \rightarrow v \) and \( v \rightarrow A u \). The Neighbor Set of node \( u \) is \( N_A(u) = \{v \in V(G) : u \rightarrow A v\} \).

Definition 4 (Topology): The topology generated by an algorithm \( A \) is a directed graph \( G_A = (E(G_A), V(G_A)) \), where \( V(G_A) = V(G) \), \( E(G_A) = \{(u, v) : u \rightarrow A v, u, v \in V(G_A)\} \).

\(^1\)By symmetric we mean that both the sender and the receiver should observe the same channel properties such as interference, path loss, and fading.

Definition 5 (Radius): The radius, \( r_u \), of node \( u \) is defined as the distance between node \( u \) and its farthest neighbor (in terms of Euclidean distance), i.e., \( r_u = \max_{v \in N_A(u)}\{d(u, v)\} \).

Definition 6 (Connectivity): For any topology generated by an algorithm \( A \), node \( u \) is said to be connected to node \( v \) (denoted \( u \Rightarrow v \)) if there exists a path \((p_0 = u, p_1, \ldots, p_{m-1}, p_m = v)\) such that \( p_i \rightarrow A p_{i+1}, i = 0, 1, \ldots, m - 1 \), where \( p_k \in V(G_A), k = 0, 1, \ldots, m \). It follows that \( u \Rightarrow v \) if \( u \Rightarrow p \) and \( p \Rightarrow v \) for some \( p \in V(G_A) \).

Definition 7 (Bi-Directionality): A topology generated by an algorithm \( A \) is bi-directional, if for any two nodes \( u, v \in V(G_A) \), \( u \in N_A(v) \) implies \( v \in N_A(u) \).

Definition 8 (Bi-Directional Connectivity): For any topology generated by an algorithm \( A \), node \( u \) is said to be bi-directionally connected to node \( v \) (denoted \( u \leftrightarrow v \)) if there exists a path \((p_0 = u, p_1, \ldots, p_{m-1}, p_m = v)\) such that \( p_i \rightarrow A p_{i+1}, i = 0, 1, \ldots, m - 1 \), where \( p_k \in V(G_A), k = 0, 1, \ldots, m \). It follows that \( u \leftrightarrow v \) if \( u \leftrightarrow p \) and \( p \leftrightarrow v \) for some \( p \in V(G_A) \).

Deriving network topology consisting of only bi-directional links facilitates link level acknowledgment, which is a critical operation for packet transmissions and retransmissions over unreliable wireless media. Bi-directionality is also important in floor acquisition mechanisms such as the RTS/CTS mechanism in IEEE 802.11.

Definition 9 (Addition and Removal): The Addition operation is to add an extra edge \((v, u)\) into \( G_A \) if \((u, v) \notin E(G_A)\), \((v, u) \notin E(G_A)\), and \( d(u, v) \leq r_u \). The Removal operation is to delete any edge \((u, v)\) from \( E(G_A) \) if \((v, u) \notin E(G_A) \) and \( d(u, v) \leq r_u \).

Both the Addition and Removal operations attempt to create a bi-directional topology by removing uni-directional edges or converting uni-directional edges into bi-directional. The resulting topology after Removal is always bi-directional, although it may be disconnected. The resulting topology after Addition is not necessarily bi-directional, as it essentially tries to increase the transmission power of a node \( v \) to a level that may be beyond its capability.

III. RELATED WORK AND WHY THEY CANNOT BE DIRECTLY APPLIED TO HETEROGENEOUS NETWORKS

Several topology control algorithms [3]–[10] have been proposed. In this section, we first summarize these algorithms and then give examples on why they cannot be directly applied to heterogeneous networks.

A. Related Work

Rodoplu et al. [4] (denoted R&M) introduced the notion of relay region and enclosure for the purpose of power control. Instead of transmitting directly, a node chooses to relay through other nodes if less power is consumed. It is shown that the network is strongly connected if every node maintains links with the nodes in its enclosure and the resulting topology is a minimum power topology. The major drawback is that it requires an explicit propagation channel model to compute the relay region. (In the simulation study presented in Section VI, we assume that the two-ray ground model is
proven to preserve network connectivity if \( p \) finds the minimum power used per node while maintaining the (bi)connectivity of the network. They introduced two algorithms to minimize the maximal power used per node while maintaining network connectivity, the traffic carrying capacity of the network is maximized, the battery life is extended, and the MAC-level contention is mitigated. The major drawback is its significant message overhead, since each node has to run multiple daemons, each of which has to exchange link state information with their counterparts at other nodes.

Some optimization methods (that are applied after the topology is derived under the base algorithm) are also discussed to further reduce the transmitting power.

To facilitate the following discussion, the definition of the Relative Neighborhood Graph (RNG) is given below.

**Definition 10 (Neighbor Relation in RNG):** For RNG [13], [14], \( (u, p) \stackrel{\text{RNG}}{\rightarrow} v \) if and only if there does not exist a third node \( p \) such that \( w(u, p) < w(u, v) \) and \( w(p, v) < w(u, v) \). Or equivalently, there is no node inside the shaded area in Fig. 1(a).

Borbash and Jennings [8] proposed to use RNG for the topology initialization of wireless networks. Based on the local knowledge, each node makes decisions to derive the network topology based on RNG. The network topology thus derived has been reported to exhibit good overall performance in terms of power usage, low interference, and reliability.

Li et al. [9] presented the Localized Delaunay Triangulation, a localized protocol that constructs a planar spanner of the Unit Disk Graph (UDG). The topology contains all edges that are both in the unit-disk graph and the Delaunay triangulation of all nodes. It is proved that the shortest path in this topology between any two nodes \( u \) and \( v \) is at most a constant factor of the shortest path connecting \( u \) and \( v \) in UDG. However, the notion of UDG and Delaunay triangulation cannot be directly extended to heterogeneous networks.

In [10], we proposed LMST (Local Minimum Spanning Tree) for topology control in homogeneous wireless multi-hop networks. In this algorithm, each node builds its local minimum spanning tree independently and only keeps one-hop nodes that are one-hop away as its neighbors in the final topology. It is proved that (1) the topology derived under LMST preserves the network connectivity; (2) the node degree of any node in the resulting topology is bounded by 6; and (3) the topology can be transformed into one with bi-directional links (without impairing the network connectivity) after removal of all uni-directional links. Simulation results show that LMST can increase the network capacity as well as reduce the energy consumption.

Instead of adjusting the transmission power of individual devices, there also exist other approaches to generate power-efficient topology. By following a probabilistic approach, Santi et al. derived the suitable common transmission range which preserves network connectivity, and established the lower and upper bounds on the probability of connectedness [15]. In [16], a “backbone protocol” is proposed to manage large wireless ad hoc networks, in which a small subset of nodes is selected to construct the backbone. In [17], a method of calculating the power-aware connected dominating sets was proposed to establish an underlying topology for the network.

**B. Why Existing Algorithms Cannot be Directly Applied to Heterogeneous Networks**

All topology control algorithms, except [4], assume homogeneous wireless nodes with uniform transmission ranges. When directly applied to heterogeneous networks, these algorithms may render disconnectivity. In this subsection, we
Fig. 2. An example that shows the optimization in CBTC ($\frac{\pi}{3}$) may lead to disconnectivity. An arrow from node $v_i$ to node $v_j$ indicates that $v_i$ can reach $v_j$. There is no path from $v_7$ to $v_1$ due to the loss of edge ($v_8$, $v_1$), which is discarded during the optimization phase since there is a shorter edge ($v_8$, $v_7$) satisfying $\angle v_7 v_8 v_1 < \frac{\pi}{3}$.

Fig. 3. An example that shows CBTC ($\frac{\pi}{3}$) without optimization may also render disconnectivity in heterogeneous networks. There is no path from $v_5$ to $v_7$ due to the loss of edge ($v_3$, $v_7$), which is discarded by $v_3$ since $v_2$, $v_4$ and $v_8$ have already provided the necessary coverage.

Fig. 4. An example that shows RNG may render disconnectivity in heterogeneous networks. There is no path from $v_7$ to $v_8$ due to the loss of edge ($v_3$, $v_8$), which is discarded since $|v_3, v_7| < |v_3, v_8|$ and $|v_8, v_7| < |v_3, v_8|$.
give several examples and motivate the need for new localized topology control algorithms.

We first give an example in Fig. 2 (a)-(c) that shows the optimization phase in CBTC(\(\frac{3}{4}\pi\)) [6] may lead to disconnectivity (note that in Figs. 2-4 we use an arrow to represent a link from u to v). As a matter of fact, as shown in Fig. 3 (a)-(b) the network topology derived under CBTC(\(\frac{3}{4}\pi\)) without optimization may still be disconnected, when the algorithm is directly applied to a heterogeneous network.

Similarly we show in Fig. 4 (a)-(b) that the network topology derived under RNG may be disconnected when the algorithm is directly applied to a heterogeneous network. As RNG is defined for undirected graphs, one may tailor the definition of RNG for directed graphs. One natural extended definition is given below.

**Definition 11 (Neighbor Relation in MRNG):** For Modified Relative Neighborhood Graph (MRNG), \(u \stackrel{MRNG}{\rightarrow} v\) if and only if there does not exist a third node \(p\) such that \(w(u, p) < w(u, v), d(u, p) \leq r_u\) and \(w(p, v) < w(u, v), d(p, v) \leq r_v\) (Fig. 1(b)).

As shown in Fig. 5 (a)-(b), the topology derived under MRNG may still be disconnected (We will give another variation of RNG for directed graphs in the next section).

**IV. DRNG AND DLMST**

In this section, we propose two localized topology control algorithms for heterogeneous wireless multi-hop networks with non-uniform transmission ranges: Directed Relative Neighborhood Graph (DRNG) and Directed Local Minimum Spanning Tree (DLMST). In both algorithms, the topology is derived by having each node build its neighbor set and adjust its transmission power based on locally collected information.

Several nice properties of both algorithms will be discussed in Section V.

Both algorithms are composed of three phases:

1) **Information Collection:** Each node collects the local information of neighbors such as position and id, and identifies the Reachable Neighborhood \(N^R\).

2) **Topology Construction:** Each node defines (in compliance with the algorithm) the proper list of neighbors for the final topology using the information in \(N^R\).

3) **Construction of Topology with Only Bi-Directional Links** (Optional): Each node adjusts its list of neighbors to make sure that all the edges are bi-directional.

a) **Information collection:** The information needed by each node \(u\) for topology control is the information of its reachable neighborhood \(N^R\). This can be obtained locally, in the case of homogeneous networks, by having each node broadcast periodically a Hello message using its maximal transmission power. The information contained in a Hello message should at least include the node id and the position of the node. These periodic messages can be sent either in the data channel or in a separate control channel. In heterogeneous networks, having each node broadcast a Hello message using its maximal transmission power may be insufficient. For example, as shown in Fig. 6, \(v_1\) is unable to know the position of \(v_4\) since \(v_4\) cannot reach \(v_1\). We will treat this issue rigorously in Section V-D. For the time being, we assume that by the end of the first phase every node \(u\) obtains its \(N^R_u\).

b) **Topology construction:** First we define the neighbor relation used in both algorithms.

**Definition 12 (Neighbor Relation in DRNG):** For Directed Relative Neighborhood Graph (DRNG), \(u \stackrel{DRNG}{\rightarrow} v\) if and only if \(d(u, v) \leq r_u\) and there does not exist a third node \(p\) such that \(w(u, p) < w(u, v)\) and \(w(p, v) < w(u, v), d(p, v) \leq r_p\) (see Fig. 1(c)).

**Definition 13 (Neighbor Relation in DLMST):** For Directed Local Minimum Spanning Tree Graph (DLMST), \(u \stackrel{DLMST}{\rightarrow} v\) if and only if \((u, v) \in E(T_u)\), where \(T_u\) is the directed local MST rooted at \(u\) that spans \(N^R_u\). That is, node \(v\) is a neighbor of node \(u\) if and only if node \(v\) is on node \(u\)’s directed local MST \(T_u\), and is “one-hop” away from node \(u\).

In the topology construction phase of DLMST, each node \(u\) computes a directed MST that spans \(N^R_u\) and takes on-tree nodes that are one hop away as its neighbors. The algorithm to compute a directed MST was first proposed by Chu and Liu.

Fig. 5. An example that shows MRNG may render disconnectivity in heterogeneous networks. There is no path from \(v_7\) to \(v_8\) due to the loss of edge \((v_7, v_8)\), which is discarded since \(|(v_1, v_7)| < |(v_1, v_8)|\), and \(|(v_8, v_7)| < |(v_1, v_8)|\).
Fig. 6. An example that shows having each node broadcast a Hello message using its maximal transmission power may be insufficient for some nodes (e.g., node \( v_1 \)) to know their reachable neighborhood. This figure also serves to show that given an arbitrary direct graph, it may be impossible to derive a bi-directional topology.

[18], and was reinvented by Edmonds [19] and Bock [20]. An efficient implementation was given by Tarjan [21] (see also [22]), which is \( O(e \log v) \) in the worse case, \( O(v \log^2 v + e) \) on average, and can be modified to be \( O(v^2) \), where \( v \) is the number of nodes and \( e \) is the number of edges in \( G_u^R \).

Each node can broadcast its own maximal transmission power in the Hello message. By measuring the receiving power of Hello messages, each node \( u \) can determine the specific power level required to reach each of its neighbors [10]. Node \( u \) then uses the power level that can reach its farthest neighbor as its transmission power. This approach can be applied to any propagation channel model.

c) Construction of topology with only bi-directional edges: As illustrated in the previous section, some links in \( G_{DLMST} \) may be uni-directional. There are two possible solutions: one can (1) enforce all the uni-directional links in \( G_{DLMST} \) to become bi-directional; or (2) delete all the uni-directional links in \( G_{DLMST} \). We will discuss these solutions in Section V-B.

V. PROPERTIES OF DRNG AND DLMST

In this section, we discuss the connectivity, bi-directionality and degree bound of DLMST, L-MST and DRNG. We always assume \( G \) is strongly connected, i.e., \( u \Rightarrow v \) in \( G \) for any \( u, v \in V(G) \).

A. Connectivity

**Lemma 1:** For any edge \((u, v) \in E(G) - E(G_{DLMST})\), let \( P = (p_0 = u, p_1, p_2, \ldots, p_{m-1}, p_m = v) \) \( (p_i, p_{i+1}) \in E(T_u) \) \( i = 0, 1, \ldots, m - 1 \) be the unique path from \( u \) to \( v \) on \( T_u \), then we have \( w(p_{m-1}, v) < w(u, v) \).

**Proof:** We prove by contradiction. Suppose \( w(p_{m-1}, v) > w(u, v) \), we can construct another directed spanning tree \( T_u' \) rooted at \( u \) with less weight, by replacing edge \((p_{m-1}, v) \) with \((u, v)\) and keeping all the other edges in \( T_u \) unchanged. This contradicts to the assumption that \( T_u \) is the local directed MST.

**Lemma 2:** Let \( T \) be the global directed MST of \( G \) rooted at any node \( w \in V(G) \), then \( E(T) \subseteq E(G_{DLMST}) \).

**Proof:** For any edge \((u, v) \in E(T)\), we prove by contradiction. Suppose \((u, v) \not\in E(G_{DLMST})\). Since \( v \) is on the directed local MST \( T_u \), there exists a unique path \( (p_0 = u, p_1, p_2, \ldots, p_{m-1}, p_m = v) \) from \( u \) to \( v \), where \( (p_i, p_{i+1}) \in E(T_u) \), \( i = 0, 1, \ldots, m - 1 \). We have \( w(p_{m-1}, v) < w(u, v) \) by Lemma 1. By replacing edge \((u, v)\) with \((p_{m-1}, v)\) and keeping all the other edges in \( T \) unchanged, we can construct another global directed spanning tree \( T' \) rooted at \( w \) that has a less weight than \( T \). This contradicts to the assumption that \( T \) is the global MST rooted at \( w \).

**Theorem 1 (Connectivity of DLMST):** If \( G \) is strongly connected, then \( G_{DLMST} \) is also strongly connected.

**Proof:** For any two nodes \( u, v \in V(G) \), there exists a unique global MST \( T \) rooted at \( u \) since \( G \) is strongly connected. Since \( E(T) \subseteq E(G_{DLMST}) \) by Lemma 2, there is a path from \( u \) to \( v \) in \( G_{DLMST} \).

**Lemma 3:** For any edge \((u, v) \in E(G)\), we have \( u \Rightarrow v \) in \( G_{DRNG} \).

**Proof:** Let all the edges \((u, v) \in E(G)\) be sorted in the increasing order of \( w(u, v) \), i.e., \( w(u_1, v_1) < w(u_2, v_2) < \ldots < w(u_l, v_l) \), where \( l \) is the total number. We prove by induction.

1) **Basis:** The first edge \((u_1, v_1)\) satisfies \( w(u_1, v_1) = \min_{(u,v) \in E(G)} \{w(u,v)\} \). We have \( d(u_1, v_1) \leq r_{\min} \), otherwise \( G \) cannot be strongly connected. For any third node \( p \), we have \( w(u, p) > w(u, v) \) and \( w(v, p) > w(v, u) \). By definition, \( u_1 \Rightarrow v_1 \) in \( G_{DRNG} \).

2) **Induction:** Assume the hypothesis holds for all edges \((u_i, v_i), 1 \leq i \leq k \), we prove \( u_k \Rightarrow v_k \) in \( G_{DRNG} \). If \( u_k \Rightarrow v_k \) in \( G_{DRNG} \), then \( u_k \Rightarrow v_k \). Otherwise, there exists a third node \( p \) such that \( w(u_k, p) < w(u_k, v_k), d(p, u_k) \leq r_{u_k} \) and \( w(p, v_k) < w(u_k, v_k), d(p, v_k) \leq r_p \). Since \((u_k, p)\) and \((p, v_k)\) are edges in \( E(G) \) with less weight than \((u_k, v_k)\), we can apply the induction hypothesis to both edges. We have \( u_k \Rightarrow p \) and \( p \Rightarrow v_k \), thus \( u_k \Rightarrow v_k \) in \( G_{DRNG} \).

**Theorem 2 (Connectivity of DRNG):** If \( G \) is strongly connected, then \( G_{DRNG} \) is also strongly connected.

**Proof:** For any two nodes \( u, v \in V(G) \), since \( G \) is strongly connected, there exists a path \((p_0 = u, p_1, p_2, \ldots, p_{m-1}, p_m = v)\) from \( u \) to \( v \), such that \((p_i, p_{i+1}) \in E(G)\), \( i = 0, 1, \ldots, m - 1 \). Thus \( p_i \Rightarrow p_{i+1} \) in \( G_{DRNG} \) by Lemma 3. Therefore, \( u \Rightarrow v \) in \( G_{DRNG} \). Hence we can conclude that \( G_{DRNG} \) is strongly connected.

B. Bi-directionality

Now we discuss the bi-directionality property of DRNG and DLMST. Since *Addition* may not always result in bi-directional topologies, we first apply *Removal* to topologies by
DLMST and DRNG. It turns out the simple Removal operation may lead to disconnectivity. Examples are given in Figs. 7–8 to show, respectively, that DLMST and DRNG with Removal may result in disconnectivity.

In general, $G$ may not be bi-directional if the transmission ranges are non-uniform. Since the maximal transmission range can not be increased, it may be impossible to find a bi-directional connected subgraph of $G$ for some cases. An example is given in Fig. 6: $v_1$ can reach $v_2$ and $v_4$, $v_2$ can reach $v_1$ and $v_3$, $v_3$ can reach $v_2$ and $v_4$, and $v_4$ can reach $v_2$ only. Addition does not lead to bi-directionality since all edges entering or leaving $v_4$ are uni-directional with all nodes already transmitting with their maximal power. On the other hand, Removal will partition the network. In this example, although the graph $G$ is strongly connected, its subgraph with the same vertex set cannot be both connected and bi-directional.

Now we show that bi-directionality can be ensured if the original topology is both strongly connected and bi-directional.

**Lemma 4:** If an edge $(u_0, v_0) \in E(G)$ satisfies $w(u_0, v_0) = \min_{(u,v) \in E(G)} \{w(u,v)\}$, then $u_0$ is a neighbor of $v_0$ in $G_{DLMST}$. $u_0$ and $v_0$ are neighbors of each other in $G_{DLMST}$.

**Proof:** We prove by contradiction. Assume $v_0$ is not a neighbor of $u_0$ in $G_{DLMST}$. We have $d(u_0, v_0) \leq r_{\text{min}}$, otherwise $G$ cannot be strongly connected. Thus $d(u_0, v_0) \leq r_{u_0}$, $d(v_0, u_0) \leq r_{v_0}$, which means $v_0 \in N_{r_{u_0}}^{u_0}$ and $u_0 \in N_{r_{v_0}}^{v_0}$. Consequently, $v_0$ is on the directed local MST $T_{u_0}$ rooted at $u_0$. Now we find the edge $(p, v_0) \in E(T_{u_0})$ that is incident to $v_0$, $p \neq u_0$ by our assumption. Since $w(p, v_0) > w(u_0, v_0)$, replacing $(p, v_0)$ with $(u_0, v_0)$ will result in a new directed local spanning tree $T'_{u_0}$ with a smaller cost than $T_{u_0}$, which is a contradiction. Therefore, $u_0 \not\rightarrow v$ in $G_{DLMST}$. It can also be proved that $v_0 \not\rightarrow u_0$ using similar arguments. Thus we have $u_0 \not\rightarrow v$ in $G_{DLMST}$.

**Lemma 5:** If the original topology $G$ is strongly connected and bi-directional, then any edge $(u, v) \in E(G)$ satisfies that $u \leftrightarrow v$ in $G_{DLMST}$.

**Proof:** For all the node pairs $(u, v) : (u, v) \in E(G)$, let them be sorted in the increasing order of $w(u, v)$, i.e., $w(u_1, v_1) < w(u_2, v_2) < \ldots < w(u_l, v_l)$ where $l$ is the total number. We prove by induction.

1) **Basis:** The first pair $(u_1, v_1)$ satisfies $w(u_1, v_1) = \min_{(u,v) \in E(G)} \{w(u,v)\}$. Thus $u_1 \not\rightarrow v_1$ by Lemma 4, which means $u_1 \leftrightarrow v_1$ in $G_{DLMST}$.

2) **Induction:** Assume the hypothesis holds for all pairs $(u_i, v_i), i < k$, we prove $u_k \leftrightarrow v_k$. If $u_k \not\rightarrow v_k$, then $u_k \not\rightarrow v_k$. Otherwise without loss of generality, we assume that $v_k$ is not a neighbor of $u_k$’s in $G_{DLMST}$. Thus $v_k$ is on the directed local MST $T_{u_k}$ and there exists a unique path $(p_0 = u_k, p_1, p_2, \ldots, p_m-1, p_m = v_k)$ from node $u_k$ to node $v_k$, where $(p_i, p_{i+1}) \in E(T_{u_k}), i = 0, 1, \ldots, m - 1$. Given that $T_{u_k}$ is the
unique local MST rooted at \( u_k \), we have \( w(p_i, p_{i+1}) < w(u_k, v_k) \), since otherwise we can construct another directed spanning tree with a less weight by replacing \((p_i, p_{i+1})\) with \((u_k, v_k)\), \((p_j, p_{j+1})\) with \((p_j, p_j)\) for all \( k \leq j \leq m - 1 \), and keeping all the other edges in \( T_u \) unchanged. Applying the induction hypothesis to each edge \((p_i, p_{i+1}), i = 0, 1, \ldots, m - 1\), we have \( p_i \Leftrightarrow p_{i+1} \), thus \( u_k \Leftrightarrow v_k \) in \( G_{\text{DLMST}} \).

**Theorem 3:** If the original topology \( G \) is strongly connected and bi-directional, then \( G_{\text{DLMST}} \) and \( G_{\text{DRNG}} \) are also strongly connected and bi-directional after Addition or Removal.

**Proof:** For any two nodes \( u, v \in V(G) \), there exists at least one path \( p = (w_0 = u, w_1, w_2, \ldots, w_{m-1}, w_m = v) \) from \( u \) to \( v \), where \( (w_i, w_{i+1}) \in E(G), i = 0, 1, \ldots, m - 1 \). Since \( w_i \Leftrightarrow w_{i+1} \) in \( G_{\text{DLMST}} \) by Lemma 5, we have \( u \Leftrightarrow v \) in \( G_{\text{DLMST}} \). Also in the proof of Lemma 3, we are only able to prove \( u_k \Rightarrow v_k \) because edge \((v_k, u_k)\) may not exist. Given \( G \) is bi-directional, we should be able to prove that \( u_k \Leftrightarrow v_k \). Therefore, \( w_i \Leftrightarrow w_{i+1} \) in \( G_{\text{DRNG}} \), which means \( u \Leftrightarrow v \) in \( G_{\text{DRNG}} \). The same results still hold after Addition or Removal, since all links in \( p \) are bi-directional and the removal of unidirectional links does not affect the existence of such a path.

**C. Degree Bound**

It has been observed that any minimum spanning tree of a simple undirected graph in the plane has a maximum node degree of 6 [23]. However, this bound does not hold for directed graphs. An example is shown in Fig. 10, where node \( u \) has 18 neighbors. In this section, we will discuss the node degree in the topology by DLMST and DRNG.

**Definition 14 (Disk):** \( \text{Disk}(u, r) \) is the disk centered at node \( u \) with a radius of \( r \).

**Definition 15 (Cone):** \( \text{Cone}(u, \alpha, v) \) is the unbounded shaded region shown in Fig. 9.

**Lemma 6:** Given three nodes \( u, v, w \in V(G_{\text{DLMST}}) \) satisfying \( w(u, v) > w(u, w) \) and \( w(u, v) > w(w, v) \), \( d(u, v) \leq r_u \), then \( u \Leftrightarrow v \) in \( G_{\text{DLMST}} \).

**Proof:** We only need to consider the case when \( d(u, v) \leq r_u \) since \( d(u, v) > r_u \) would imply \( u \rightarrow v \). Assume \( u \rightarrow v \). Since \( d(u, w) \leq d(u, v) \leq r_u \), there exists a unique path \( p = (v_0 = u, v_1, v_2, \ldots, v_{m-1}, v_m = w) \) on \( T_u \) from node \( u \) to node \( w \), where \((v_i, v_{i+1}) \in E(T_u), i = 0, 1, \ldots, m - 1 \). If \( v \) is on the path \( p \), replacing edge \((u, v)\) with edge \((u, w)\) and keeping all other edges unchanged in \( T_u \) will result in a spanning tree of \( G_u \) with a smaller weight. If \( v \) is not on \( p \), replacing edge \((u, v)\) with edge \((u, w)\) and keeping all other edges unchanged in \( T_u \) will result in a spanning tree of \( G_u \) with a smaller weight. Both scenarios contradict with the fact that \( T_u \) is the unique minimum spanning tree of \( G_u \).

**Corollary 1:** If \( v \) is a neighbor of \( u \)’s in \( G_{\text{DLMST}} \), and \( d(u, v) \geq r_{\min} \), then \( u \) can not have any other neighbor inside \( \text{Disk}(v, r_{\min}) \).

**Theorem 4:** For any node \( u \in V(G_{\text{DLMST}}) \), the number of neighbors in \( G_{\text{DLMST}} \) that are inside \( \text{Disk}(u, r_{\min}) \) is at most 6.

**Proof:** Let \( N(u) \) be the set of neighbors of \( u \) in \( G_{\text{DLMST}} \) that are inside \( \text{Disk}(u, r_{\min}) \). Let the nodes in \( N(u) \) be ordered such that for the \( i \)th node \( w_i \) and the \( j \)th node \( w_j \) (\( j > i \)), \( w(u, w_j) > w(u, w_i) \). By Lemma 6, we have \( w(u, v_j) \leq w(w_i, v_j) \) (otherwise \( u \rightarrow w_j \)). Thus \( \angle v_i w_i v_j \geq \pi/3 \), i.e., node \( w_j \) can not reside inside \( \text{Cone}(u, 2\pi/3, w_i) \). Therefore, node \( u \) can not have neighbors other than node \( w_i \) inside \( \text{Cone}(u, 2\pi/3, w_i) \). By induction on the rank of nodes in \( N(u) \), the maximal number of neighbors that \( u \) can have is at most 6.

**Theorem 5:** The out degree of node in \( G_{\text{DLMST}} \) is bounded by a constant that depends only on \( r_{\max} \) and \( r_{\min} \).

**Proof:** For any node \( u \) in \( G_{\text{DLMST}} \), there are at most 6 neighbors inside \( \text{Disk}(u, r_{\min}) \) from Theorem 4. Also from Corollary 1, the set of disks \( \{ \text{Disk}(v, r_{\min}) : v \in N_{\text{DLMST}}(u), v \notin \text{Disk}(u, r_{\min}) \} \) are disjoint. Therefore, the
the hexagonal area (as shown in Fig. 10) centered at every ϵ
if
the scenario where the maximum out degree of β
transmitting to a specific neighbor, node
(except in some extreme cases, e.g., a large number of nodes
bounded for an arbitrary topology. However, with the help
directional antennas are used, the physical degree cannot be
the number of nodes within the transmission radius. If omni-
logical
Also note that what has been discussed so far is actually the
is much smaller for networks with random distributed nodes.
will show in Section VI that the average maximum degree
it is more important to consider the
r
shaded area, as long as
example is given in Fig. 11. For all
G
DRNG
have different maximal transmission powers, the operation of
all the other nodes within r_u is not sufficient to ensure each
node u obtains the information of reachable neighborhood N_u^R
(Fig. (6)). Fortunately with the desirable properties of DRNG
and DLMST proved in Sections V-A and V-B, we show that it
is sufficient for node u to collect neighborhood information
only from nodes whose maximal transmission range covers
node u. That is, the original information exchange algorithm
that requires only “one-hop” information suffices.

Consider a directed simple graph with less edges: G' =
(V(G'), E(G')), where E(G') = \{ (u, v) : d(u, v) ≤
min(r_u, r_v), u, v ∈ V(G) \}. For any edge (u, v) ∈ E(G'),
since d(u, v) ≤ min(r_u, r_v), we have (v, u) ∈ E(G'), which
means G' is bi-directional. Define N_u'^R = \{ v ∈ V(G) :
d(u, v) ≤ min(r_u, r_v) \}, r_u' = max_v∈N_u'^R \{d(u, v)\},
where r_u' ≤ r_u since for any v ∈ N_u'^R, d(u, v) ≤ r_u. Let r_min' =
min_v∈V \{r_v'\} and r_max' = max_v∈V \{r_v'\}. By requiring each
node u to broadcast its position and id to all other nodes within
r_u, we are able to determine N_u'^R and r_u'. We can then apply
DRNG and DLMST on top of G' and prove that Theorems
1–4 still hold even if the original topology is G'.

**Theorem 6**: Theorems 1–5 still holds if the original topo-
ogy is G'.

**Proof**: We replace G, r_u, N_u^R, r_min and max with G',
r_u', N_u'^R, r_min' and r_max' in the proof of Lemma 1–6 and
Theorem 1–5. Then following the same line of arguments, we
can prove that they still hold if the original topology is G'.

**Theorem 7**: If the original topology is G' (which is a subgraph of G), G_DLMST and G_DRNG are bi-directional after
Addition or Removal.

**Proof**: We apply Theorem 3 to G', as G' is bi-directional.

**VI. Simulation Study**

In this section, we evaluate the performance of R&M,
DRNG, and DLMST by simulations. All three algorithms
are known to preserve network connectivity in heterogeneous
networks.

In the first simulation, 50 nodes are uniformly distributed
in a 1000m × 1000m region. The transmission ranges for
nodes are uniformly distributed in [200m, 250m]. Fig. 12
shows the topologies derived using the maximal transmission power
(labeled as NONE), R&M (under the two-ray ground model),
DRNG, and DLMST for one simulation instance. As shown in
Fig. 12, R&M, DRNG and LMST all significantly reduce the
average node degree, while maintaining network connectivity.
Moreover, both DRNG and DLMST outperform R&M in the
sense that fewer edges are formed in the topology.

In the second simulation, we vary the number of nodes in
the region from 80 to 300, and each data point is an average
of 100 simulation runs. The transmission ranges of nodes are
uniformly distributed in [10m, 250m]. Fig. 13 shows the
average radius and the average edge length for the topologies
derived under NONE (no topology control), R&M, DRNG, and
DLMST. DLMST outperforms the others, which implies that
DLMST can provide a better spatial reuse and use less energy
to communicate.
We also compare the out degree of the topologies by different algorithms. The result of NONE is not shown because the out degrees increase almost linearly with the number of nodes and are significantly larger than those under R&M, DRNG, and DLMST. Fig. 14 shows the average logical/physical out degree for the topologies derived by R&M, DRNG, and DLMST. The average out degrees under R&M and DRNG increase with the increase in the number of nodes, while those under DLMST actually decrease. Fig. 15 shows the average maximum logical degree and the largest maximum logical out degree for each number of nodes. The largest maximum logical degree under DLMST is at most 4, and is well below the theoretical upper bound obtained in Theorem 5. Also DLMST has much smaller degrees than the other topologies. Similar results can be observed in Fig. 16 for physical degrees. The only difference is that the physical degrees are in general much higher than the logical degrees for the same network.

VII. CONCLUSIONS

In this paper, we have proposed two local topology control algorithms, Directed Relative Neighborhood Graph (DRNG) and Directed Local Minimum Spanning Tree (DLMST), for heterogeneous wireless multi-hop networks in which each node may have different maximal transmission ranges. We show that as most existing topology control algorithms (except R&M [4]) do not consider the fact that nodes may have different maximal transmission ranges, they render disconnected network topology when directly applied to heterogeneous networks. Then we devise DRNG and DLMST and prove that (i) both DRNG and DLMST preserve network connectivity; (ii) both DRNG and DLMST preserve network bi-directionality if Addition and Remove operations are applied to the topologies.
derived under these algorithms; and (iii) the out degree of any node is bounded in the topology derived under DLMST, while that may be unbounded under DRNG. The simulation study validates the superiority of DRNG and DLMST over R&M.

As part of our future research, we will pursue the following open problems: (1) given a topology in which each node transmits with different maximal transmission power, what is the probability that the topology is bi-directional with respect to the distribution and the density of nodes, and the distribution of the transmission ranges? and (2) How will MAC-level interference affect network connectivity and bi-directionality?

REFERENCES


Fig. 13. Comparison of DLMST, DRNG and R&M with respect to average radius and average edge length.

Fig. 14. Comparison of R&M, DRNG and DLMST with respect to average out degree.
(a) Average maximum logical out degree.

(b) Largest maximum logical out degree.

Fig. 15. Comparison of R&M, DRNG and DLMST with respect to the maximum logical degree.

(a) Average maximum physical out degree.

(b) Largest maximum physical out degree.

Fig. 16. Comparison of R&M, DRNG and DLMST with respect to the maximum physical degree.


