Topology Control in Heterogeneous Wireless Ad-hoc Networks

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Overview of this Presentation

- Part 1:
  - Introduction

- Part 2:
  - Paper 1: General Graphs
    - DRNG & DLMST
  - Paper 2: Mutual Inclusion Graphs
    - $EYG_k(MG)$

- Part 3:
  - Proof for connectivity in DLMST
  - Conclusions
About the Papers

- **Topology Control in Heterogeneous Wireless Networks: Problems and Solutions**
  - Ning Li and Jennifer C. Hou
  - INFOCOM 2004

- **Localized Topology Control for Heterogeneous Wireless Ad-hoc Networks**
  - Xiang-Yang Li, Wen-Zhan Song, Yu Wang
  - MASS 2004
Part 1

Introduction
Why Topology Control?

- maintain network connectivity
  - every node can reach all others
  - reduce energy consumption
  - sending over near neighbours is more efficient than sending directly to a far target
  - do not send with maximal transmission power if not necessary

- improve network capacity
Related Work

- based on centralised algorithms
  - applicable for static networks
  - need global information
  - can achieve optimality

- based on unit disk graphs
  - homogeneous wireless nodes with uniform transmission ranges
  - every node sends with same transmission power

- based on fixed nodes
  - once a node has been initialised, it does not change its position
Why Heterogeneous Networks? (1)

- can easily add new devices without attention to the type of the device (mobility, dynamic)
  - we can use devices with non-uniform transmission ranges
- in practice there are many influences which affect the range of a device
  - obstacles like plants, walls, ... or other radio frequencies
Why Heterogeneous Networks? (2)

- there exist heterogeneous networks in which devices have dramatically different capabilities
  - Military: devices on soldiers vs. devices on vehicles
- even devices of the same type may have slightly different maximal transmission power
What we want

- each wireless node should locally
  - adjust its transmission power
  - select with which neighbours to communicate
- model should deal with dynamic changes in topology
  - addition of new nodes
  - removal or drop out of links (or nodes)
Simple adaptation doesn't work (1)

- can't guarantee network connectivity in heterogeneous case
  - no global information
  - assumptions about transmission power of counterparts don't hold anymore
- message overhead
  - energy
- unbounded out-degree
  - increase signal interference & overhead at a node
Simple adaptation doesn't work (2)

- A RNG structure in a homogeneous graph is connected since all links would be bi-directional.
- Edge $(v_3, v_8)$ is discarded since $v_7$ lies in the shaded area between $v_3$ and $v_8$. 
Part 2

General Graphs

&

Mutual Inclusion Graphs
G: (General Graph)

- a node $u$ connects to another node $v$ iff the Euclidean distance between these two nodes is smaller than the transmission range of $u$
- this model has uni- and bi-directional connections
Reachable Neighbourhood (1)

- in DRNG and DLMST each node has to know its reachable neighbourhood
  - set of nodes that a specific node can reach using its maximal transmission power (e.g. for $v_1$ we get $v_2$ and $v_4$)

![Diagram showing nodes and their connections](image)
Reachable Neighbourhood (2)

- finding this reachable neighbourhood is difficult since $v_4$ can't reach $v_1$
  - unfortunately it is not described in the paper how they will manage this in the General Graph
Directed RNG (Relative Neighbourhood Graph)

- **Algorithm:**
  - Collect reachable neighbourhood
  - Build topology by selecting those nodes from the reachable neighbourhood for which there does not exist a node $p$ that is closer to $u$ and $v$ than $u$ to $v$ and $p$ can reach $v$. 
Directed Local MST (Minimum Spanning Tree)

Algorithm:
- collect reachable neighbourhood
- build topology computing a directed MST for each node that spans the reachable neighbourhood of this node and takes on-tree nodes that are one hop away as its neighbours.
MG: (Mutual Inclusion Graph)

- two nodes are connected iff they are within the maximum transmission range of each other
- there are only bi-directional links
Planar Topology

- for any topology control method it is not always possible to create a planar topology while keeping the communication graph connected
  - u is out of the transmission range of x and y, while v is in the transmission range of y and out of the range of x
  - according to MG, there are only xy, vy and uv in the graph
Sparse Structure

- based on RNG they found an extension that has bounded number of links $\rightarrow$ sparse structure
- unfortunately that's not what we want
- we are looking for bounded out-degree
Idea of Spanners

- Given a graph $G$ and a subgraph $H$ of $G$.
- $H$ is a $t$-Length Spanner of $G$ if for any two nodes $u,v \in V(H)$ the shortest path between $u$ and $v$ is at most a constant factor $t$ longer than the shortest path of these two nodes in $G$.
- if the weighting function is not the length but the power than we have with the same argumentation a Power Spanner instead of a Length Spanner
Power Spanner

- based on GG (Gabriel Graph) they found a graph which contains the minimum power consumption path for any two nodes in MG
- we are still looking for bounded out degree
Degree-Bounded Spanner (1)

- based on Yao Graph
- at each node $u$, partition space into $k$ equal subspaces (= cones) and connect to the nearest node in each cone if there is any
Degree-Bounded Spanner (2)

- in a MG model simply selecting the closest incoming neighbour does not guarantee connectivity
  - $v, w$ are in same cone of $u$; $x, u$ are in same cone of $v$
  - node $u$ keeps link $uw$ and node $w$ keeps link $uw$
  - node $v$ keeps link $vx$ and node $x$ keeps link $xv$. 
Novel Space Partition

- partition space into $k$ equal subspaces (= cones)
- divide each cone into constant number of subsets and connect $v$ to the nearest node $w$ in each subset
- the algorithm guarantees that all nodes in a subset are connected to node $w$ in this subset
$EYG_k(MG)$: (Extended Yao Graph)

- has bounded out-degree in $O(\log_2 q)$
- is a Length- and a Power-Spanner to $MG$
- is connected if $MG$ is connected
- is bi-directional
- they reach almost optimum since any connected graph will have degree at least $O(\log_2 q)$

$q = \max_{v,w} \frac{r_v}{r_w}$ with $v \in V(MG)$ and $wv \in EYG_k(MG)$
Part 3

Proof & Conclusions
Proof for connectivity in $G_{DLMST}$ (1)

- **Lemma 1:**

  For any edge $(u, v)$ which is only in $G$ but not in $G_{DLMST}$, there must be a unique path on $T_u$ from $u$ to $v$ in $G_{DLMST}$. Let $p$ be the last node on this path before $v$ than we have $w(p, v) < w(u, v)$.

  \[ w(u, v): \text{gives any edge in a graph a unique weight} \]
  \[ T_u: \text{local MST rooted at node u containing all reachable nodes of u} \]
  \[ G: \text{General Graph} \]
Proof for connectivity in $G_{DLMST}$ (2)

- Proof (by contradiction):

Suppose $w(p, v) > w(u, v)$, we can construct another directed spanning tree $T'_u$ rooted at $u$ with lower weight, by replacing edge $(p, v)$ with $(u, v)$ and keeping all the other edges in $T_u$ unchanged. This contradicts the assumption that $T_u$ is the local directed MST.
Proof for connectivity in $G_{DLMST}$ (1)

- **Lemma 2:**
  Let $T$ be the global directed MST of $G$ rooted at any node $w \in V(G)$, than $E(T) \subseteq E(G_{DLMST})$.

- **Proof (by contradiction):**
  For any edge $(u, v) \in E(T)$ suppose $(u, v) \notin E(G_{DLMST})$. Since $v$ is on the directed local MST $T_u$, there exists a unique path from $u$ to $v$ with $p$ as the last node on this path before $v$. We have $w(p, v) < w(u, v)$ by Lemma 1. By replacing edge $(u, v)$ with $(p, v)$ and keeping all the other edges in $T$ unchanged, we can construct another global directed spanning tree $T$ rooted at $w$ that has lower weight than $T$. This contradicts the assumption that $T$ is the global MST rooted at $w$. 
Proof for connectivity in $G_{DLMST}$ (4)

- **Theorem 1 (Connectivity of $G_{DLMST}$):**
  
  If $G$ is strongly connected, then $G_{DLMST}$ is also strongly connected.

- **Proof (by contradiction):**

  For any two nodes $u, v \in V(G)$, there exists a unique global MST $T$ rooted at $u$ since $G$ is strongly connected. Since $E(T) \subseteq E(G_{DLMST})$ by Lemma 2, there is a path from $u$ to $v$ in $G_{DLMST}$. 
Conclusions: Paper 1 (1)

- for a General Graph there are two localized topology control algorithms, DLMST and DRNG, which preserve connectivity.
- DLMST and DRNG preserve bi-directionality if they are based on a Mutual Inclusion Graph and Addition & Remove operations are applied.
Conclusions: Paper 1 (2)

- DLMST has a bounded out-degree while DRNG may be unbounded
- There is no description of how exactly they find the reachable neighbourhood
  - It is more like a theoretical and mathematical work showing the general possibility for building such topologies based on a General Graph
Conclusions: Paper 2

- $\text{EYG}_k(MG)$ has a stricter bound on the out-degree than DLMST and guarantees better characteristics
- Length- and Power-Spanner to MG
- they reach almost optimum since any connected graph will have degree at least $O(\log_2 q)$
Questions?