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Principles of Distributed Computing
Exercise 7: Sample Solution

1 Sorting Networks

We will assume that a comparator outputs the smaller value on the top and the larger value on the bottom wire.

a) Wrong. Consider the following input, from top to bottom: $I = (0, 0, 1, 0, 0, 0)$ which produces the output $O = (0, 0, 0, 1, 0, 0)$. This is obviously not sorted.

b) Wrong. A comparator leaves a sorted sequence intact.

c) Correct. A correct sorting network needs to be able to sort *any* input sequence, and a comparator added to the front merely changes the input for the sorting network.

d) Correct. Assume otherwise: There is a correct sorting network S such that there is no comparator between wires i and $i + 1$. Consider the input sequence I consisting of 0's, then a 1 at the ith spot, a 0 at the spot $i + 1$, and the rest 1s. Now every wire will leave I intact, because I is already sorted except for the wires i and $i + 1$. But since there is no wire, the output to S is the same as the input, $O = I$, which is not sorted. Thus, no such S exists.

e) Wrong. Consider the following network with three wires depicted in Figure 1. It does not sort $I = (1, 1, 0)$ because the 0 does not have a chance to get to the top wire, so $O = (1, 0, 1)$.

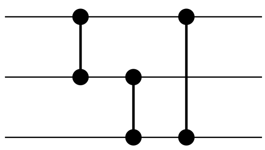

Figure 1: A network with a comparator between each pair of wires.

f) Wrong. This can be seen in two ways. The first is by a simple counter example as in Figure 2. If we leave out the marked comparator, then we have a correct sorting network (easy to see or otherwise check all 0-1 sequences). Adding it gives us the same problem as the network in Figure 1 above.

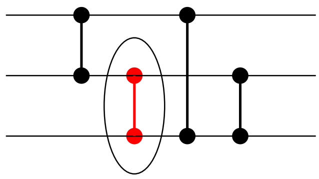

Figure 2: A network with an additional comparator added inside.

Another indication for why this statement is wrong comes from considering Batcher's Sorting Network from the lecture. It relies on creating and sorting bitonic sequences. A comparator can easily destroy a bitonic sequence, such as $(0, 1, 1, 0)$ into $(0, 1, 0, 1)$.