for integers b, a with $b > a$

$$H(b) - H(a) = \sum_{i=1}^{b} 1/i - \sum_{i=1}^{a} 1/i$$

$$= \sum_{i=a+1}^{b} 1/i \geq (b-a)\frac{1}{b}$$

$$\ldots \sum_{i=1}^{k-1} (u_i - u_{i+1}) \cdot \frac{1}{u_i}$$

$$\leq \sum_{i=1}^{k-1} H(u_i) - H(u_{i+1})$$

$$= H(u_1) - \cancel{H(u_2) + H(u_2)} - \cancel{H(u_3) + H(u_3)} - \cancel{H(u_4) + \ldots} \cancel{H(u_{k-1})} - H(u_k)$$

$$= H(u_1) - \underbrace{H(u_k)}_{=0} = H(u_1)$$

$u_0 = \Delta(u) + 1$; $u_1 < u_0 \Rightarrow u_1 \leq \Delta(u)$

done.

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