1 A Quorum System

Consider a Quorum System with 7 nodes numbered from 001 to 111, where each three nodes fulfilling \( x \oplus y = z \) constitute a quorum. In the following picture this quorum system is represented: All nodes on a line (such as 111, 010, 101) and the nodes on the circle (010, 100, 110) form a quorum.

\begin{center}
\begin{tikzpicture}
\node at (0,0) {111};
\node at (1,1) {100};
\node at (1,-1) {011};
\node at (-1,1) {110};
\node at (-1,-1) {010};
\node at (0,0) {001};
\draw (1,1) -- (1,-1) -- (-1,1) -- (-1,-1) -- (1,1);
\end{tikzpicture}
\end{center}

\textbf{a)} Of how many different quorums does this system consist and what are its work and its load?

\textbf{b)} Calculate its resilience \( f \). Give an example where this quorum system does not work anymore with \( f + 1 \) faulty nodes.

\textbf{c)} Calculate its failure probability if each node fails independently with a probability \( p \).

2 The Resilience of a Quorum System

Does a quorum system exist, which still works although all nodes of a specific quorum fail? Give an example or prove its nonexistence.

3 S-Uniform Quorum Systems

\textbf{Definitions:}

\textbf{S-uniform:} A quorum system \( Q \) is \textit{s-uniform} if every quorum in \( Q \) has exactly \( s \) elements.

\textbf{Balanced access strategy:} An access strategy \( W \) for a quorum system \( Q \) is \textit{balanced} if it satisfies \( l_W(i) = L \) for all \( P_i \in P \).

\textbf{Claim:} An \( s \)-uniform quorum system \( Q \) reaches an optimal load with a balanced access strategy.

\textbf{a)} Describe in your own words, why this claim is true.

\textbf{b)} Prove the optimality of a balanced access strategy on an \( s \)-uniform quorum system.
4 Chubby

Chubby is a distributed lock management system, which is used in different settings, such as to provide locks for the Google File System (GFS).

a) You have learned that Chubby provides coarse grained locking. What does this mean? Why did the designers of Chubby choose this approach?

b) Chubby can manage multiple locks, and it is able to store meta-data to each lock. Describe how these properties can be used to implement a reliable name-service.

c) A Chubby cell typically contains 5 servers. What would be the effect of using a smaller or larger number of servers per cell?