Cooperative Diversity by Relay Phase Rotations in Block Fading Environments

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Abstract — **We consider a wireless network with one source/destination pair and several linear amplify-and-forward relays. All nodes are equipped with one single antenna. To achieve cooperative diversity we propose time-variant and relay specific phase rotations induced at each relay to make the effective channel time-variant. This transformation of spatial diversity into temporal diversity can be exploited by an appropriate outer code. Furthermore we show that the allocation of the amplification gains at the relays has great influence on the diversity performance and we give a low complexity extension to existing gain allocations.**

I. Introduction

The use of diversity in the spatial and time dimension to mitigate the effects of fading and therefore to increase the reliability of radio links in wireless networks is a well known technique for systems with co-located antennas (space-time coding). Recently a new form of realizing spatial diversity has been introduced in [1] and [2] called *cooperative diversity* or *user cooperation diversity*. The main idea is to use multiple single antenna nodes as a virtual macro antenna array, realizing spatial diversity in a distributed fashion. In such a network several nodes serve as relays for one active source/destination pair. Relays can be classified as either *decode-and-forward* (DF) or *amplify-and-forward* (AF). AF relays, which are considered in this work, only retransmit an amplified version of their received signals. This leads to low-complexity relay transceivers, lower power consumption since there is no signal processing for decoding procedures, and is transparent to adaptive modulation techniques which may be employed by the source.

Previous works on cooperative relaying can also be found e.g. in [3] where a general information theoretic framework about relaying channels is established. In [4] a cooperation scheme for two users communicating with a base station by using existing channel coding methods is proposed. In [5] the outage and the ergodic capacity behavior of different relaying protocols is analyzed. In [6] and [7] it is shown how the capacity of ill-conditioned MIMO channels can be improved by cooperative relay nodes that act as active scatterers. A distributed implementation of the Alamouti space-time coding scheme is presented in [8]. In the distributed case this scheme is not able to make the effective channel orthogonal, but it still achieves a degree of diversity which is larger than two. Unfortunately, full rate orthogonal space-time block codes for more than two antennas which can be assigned to cooperative relay networks (more than two relays) are not available.

Consider a network with $L \geq 2$ relays with uncorrelated frequency flat channel coefficients between relays and destination in a block fading environment (static nodes topology). An appropriate code should make use of these L different random variables to achieve diversity of order L. But unfortunately all channel coefficients add up to only one effective channel coefficient which is constant within one fading interval. If the AF relays only retransmit their signals, the exploitation of time-diversity by an outer code is not possible. *Time-variant and relay specific phase rotations* at the relays result in a time-variant effective relay channel, which can be utilized by an outer code as presented in [9] or [10] to achieve diversity. These phase rotations are easy to implement and therefore lead only to a small extension in signal processing power at the relays. Our proposed system is feasible for an arbitrary number of relay nodes e.g. using the outer codes in [9] and [10].

Due to the relative position of the relays to source and destination a *near-far effect* which has an negative impact on the system performance may appear. As an example if a relay is far away from the source or in a deep fade the received SNR is low. If this relay is near to the destination, the amplified SNR will dominate the resulting SNR at the destination. Therefore a gain allocation which depends on the SNR at the relay is necessary.

Own Contribution: In this work we describe a general system model for a wireless network with one source/destination pair and several AF relays operating in a block fading environment. All nodes are equipped with one antenna. We determine the mutual information of this system model. To achieve diversity we propose time-variant and relay specific phase rotations induced at each relay to make the effective channel time-variant. This transformation of spatial diversity into temporal diversity can be utilized by an outer code and is available for an arbitrary number of relay nodes. Furthermore, we show that the allocation of the amplification gains at the relays has great influence on the diversity performance and we give a low complexity extension to existing gain allocations.

Organization of the Paper: The remainder of the paper is organized as follows. The next section introduces the system model and gives the corresponding mutual information. The idea of high order cooperative diversity by time-variant and relay specific phase rotations is presented in section III. In section IV we propose an extension of existing gain allocations to circumvent the near-far effect. Performance results are presented and discussed in section V. Conclusions are given in the last section.

Notation: We shall use bold uppercase letters to denote matrices and bold lowercase letters to denote vectors. Further $(\cdot)^T$, $(\cdot)^\dagger$ stand for transpose and Hermitian transpose of a matrix, respectively. $\mathrm{diag}[a, \ldots, z]$ denotes a diagonal matrix with the elements $a, \ldots, z$ on its main diagonal, $\mathbf{I}$ is an identity matrix and $\mathbf{0}$ a matrix with all elements equal to zero. $\mathrm{E}\{\cdot\}$ is the expectation operator. Here a circularly symmetric complex Gaussian random variable Z is a random variable $Z = X + jY \sim \mathcal{CN}(m, \sigma^2)$, where X and Y are i.i.d. $\mathcal{N}(m, \sigma^2/2)$.

II. System Model and Mutual Information

In the following we derive a linear system model for a point-to-point link in a single-antenna 2-hop cooperative relaying network depicted in Fig. 1. We consider a network consisting of a source S, a destination D, and L relays R_1 to R_L.

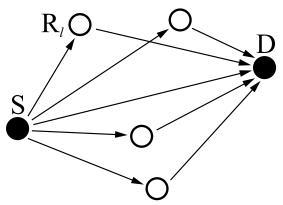

Fig. 1: 2-hop cooperative relaying network with single antenna nodes

In such a network the transmission of a data packet of size N_B symbols from the source to the destination occupies two time slots of length N_B[1]. In the first time slot the source transmits the data packet. There are L relays receiving during the first time slot and retransmitting the signal during the second time slot. The destination receives in both time slots (traffic pattern T1). Another possible traffic pattern arises if the source sends also in the second time slot (T2). If the direct link between source and destination is blocked (maybe by shadowing) the destination receives only the signals from the relays in the second slot (T3).

	T1		T2		T3	
	Slot1	Slot 2	Slot1	Slot 2	Slot1	Slot 2
Source	Tx	/	Tx	Tx	Tx	/
Relays	Rx	Tx	Rx	Tx	Rx	Tx
Destinat.	Rx	Rx	Rx	Rx	/	Rx

Tab. 1: Traffic patterns of source, relays, and destination during the two time slots

At time instance k in the first time slot the source sends the symbol s_k, with average transmitted power $\mathrm{E}\{|s_k|^2\} = P$. The received signals at the destination D and at the relay R_l are given by

$$r_k = h_k^{(\mathrm{S},\mathrm{D})} s_k + w_k \tag{1}$$

$$y_k^{(\mathrm{R}_l)} = h_k^{(\mathrm{S},\mathrm{R}_l)} s_k + w_k^{(\mathrm{R}_l)} \tag{2}$$

where $h_k^{(\mathrm{S},\mathrm{D})}$ and $h_k^{(\mathrm{S},\mathrm{R}_l)}$ are the flat fading channel coefficients which usually include path loss, shadowing and small-scale fading. The AWGN contributions are given by w_k and $w_k^{(\mathrm{R}_l)}$. For ease of notation we stack the transmitted symbols of the source in the first time slot in the vector

$$\mathbf{s}_b = \begin{bmatrix} s_k, & s_{k+1}, & \ldots, & s_{k+N_\mathrm{B}-1} \end{bmatrix}^T, \quad \mathbf{s}_b \in \mathbb{C}^{N_\mathrm{B}\times 1}.$$

The subscript b denotes the index of the time slot, where the relation between b and k (Fig. 2) is given by $b = \lfloor \frac{k}{N_\mathrm{B}} \rfloor$.

Organizing the channel coefficients in diagonal matrices $\mathbf{H}_b^{(\mathrm{S},\mathrm{D})}$ and $\mathbf{H}_b^{(\mathrm{S},\mathrm{R}_l)} \in \mathbb{C}^{N_\mathrm{B}\times N_\mathrm{B}}$ given by

$$\mathbf{H}_b^{(\mathrm{S},\mathrm{D})} = \mathrm{diag}\begin{bmatrix} h_k^{(\mathrm{S},\mathrm{D})}, & h_{k+1}^{(\mathrm{S},\mathrm{D})}, & \ldots, & h_{k+N_\mathrm{B}-1}^{(\mathrm{S},\mathrm{D})} \end{bmatrix}$$

$$\mathbf{H}_b^{(\mathrm{S},\mathrm{R}_l)} = \mathrm{diag}\begin{bmatrix} h_k^{(\mathrm{S},\mathrm{R}_l)}, & h_{k+1}^{(\mathrm{S},\mathrm{R}_l)}, & \ldots, & h_{k+N_\mathrm{B}-1}^{(\mathrm{S},\mathrm{R}_l)} \end{bmatrix}$$

[1] We assume that the number of symbols that should be transmitted is larger or equal to the number of relays $N_\mathrm{B} \geq L$.

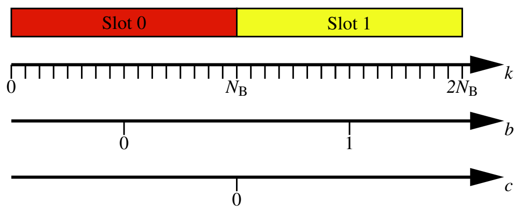

Fig. 2: Illustration of different time axis used in the system model

the received signals of destination D and of relay R_l are given by

$$\mathbf{r}_b = \mathbf{H}_b^{(\mathrm{S},\mathrm{D})} \mathbf{s}_b + \mathbf{w}_b \tag{3}$$

$$\mathbf{y}_b^{(\mathrm{R}_l)} = \mathbf{H}_b^{(\mathrm{S},\mathrm{R}_l)} \mathbf{s}_b + \mathbf{w}_b^{(\mathrm{R}_l)}. \tag{4}$$

Note that our assumption of a block fading channel leads to constant elements on the diagonal of the previous defined channel matrices.

In the second time slot $b+1$ relay R_l retransmits an amplified version of the received signals $\mathbf{y}_b^{(\mathrm{R}_l)}$. This amplification is done by a diagonal matrix $\mathbf{G}_b^{(\mathrm{R}_l)} \in \mathbb{C}^{N_\mathrm{B}\times N_\mathrm{B}}$ defined as

$$\mathbf{G}_b^{(\mathrm{R}_l)} = \mathrm{diag}\begin{bmatrix} g_k^{(\mathrm{R}_l)}, & g_{k+1}^{(\mathrm{R}_l)}, & \ldots, & g_{k+N_\mathrm{B}-1}^{(\mathrm{R}_l)} \end{bmatrix}.$$

Therefore the received signal at the destination D in the second time slot $b+1$ given by the vector $\mathbf{r}_{b+1} \in \mathbb{C}^{N_\mathrm{B}\times 1}$ can be expressed as

$$\begin{aligned} \mathbf{r}_{b+1} &= \left(\sum_{l=1}^{L} \mathbf{H}_{b+1}^{(\mathrm{R}_l,\mathrm{D})} \mathbf{G}_{b+1}^{(\mathrm{R}_l)} \mathbf{y}_b^{(\mathrm{R}_l)} \right) + \mathbf{w}_{b+1} \\ &= \left(\sum_{l=1}^{L} \mathbf{H}_{b+1}^{(\mathrm{R}_l,\mathrm{D})} \mathbf{G}_{b+1}^{(\mathrm{R}_l)} \mathbf{H}_b^{(\mathrm{S},\mathrm{R}_l)} \right) \mathbf{s}_b \\ &\quad + \sum_{l=1}^{L} \mathbf{H}_{b+1}^{(\mathrm{R}_l,\mathrm{D})} \mathbf{G}_{b+1}^{(\mathrm{R}_l)} \mathbf{w}_b^{(\mathrm{R}_l)} + \mathbf{w}_{b+1}. \end{aligned} \tag{5}$$

Stacking both received vectors $\mathbf{r}_b$ and $\mathbf{r}_{b+1}$ together in a compound vector $\mathbf{r}_c \in \mathbb{C}^{2N_\mathrm{B}\times 1}$, we can describe our system as

$$\begin{aligned} \mathbf{r}_c = \begin{bmatrix} \mathbf{r}_b \\ \mathbf{r}_{b+1} \end{bmatrix} &= \begin{bmatrix} \mathbf{H}_b^{(\mathrm{S},\mathrm{D})} \\ \sum_{l=1}^{L} \mathbf{H}_{b+1}^{(\mathrm{R}_l,\mathrm{D})} \mathbf{G}_{b+1}^{(\mathrm{R}_l)} \mathbf{H}_b^{(\mathrm{S},\mathrm{R}_l)} \end{bmatrix} \mathbf{s}_b \\ &\quad + \begin{bmatrix} \mathbf{w}_b \\ \mathbf{w}_{b+1} + \sum_{l=1}^{L} \mathbf{H}_{b+1}^{(\mathrm{R}_l,\mathrm{D})} \mathbf{G}_{b+1}^{(\mathrm{R}_l)} \mathbf{w}_b^{(\mathrm{R}_l)} \end{bmatrix}. \end{aligned} \tag{6}$$

If we furthermore assume that the source also transmits the signals $\mathbf{s}_{b+1} \in \mathbb{C}^{N_\mathrm{B}\times 1}$ in the second time slot $b+1$ our system model (traffic pattern T2) expands to

$$\begin{aligned} \mathbf{r}_c = &\begin{bmatrix} \mathbf{H}_b^{(\mathrm{S},\mathrm{D})} & \mathbf{0} \\ \sum_{l=1}^{L} \mathbf{H}_{b+1}^{(\mathrm{R}_l,\mathrm{D})} \mathbf{G}_{b+1}^{(\mathrm{R}_l)} \mathbf{H}_b^{(\mathrm{S},\mathrm{R}_l)} & \mathbf{H}_{b+1}^{(\mathrm{S},\mathrm{D})} \end{bmatrix} \begin{bmatrix} \mathbf{s}_b \\ \mathbf{s}_{b+1} \end{bmatrix} \\ &+ \begin{bmatrix} \mathbf{w}_b \\ \mathbf{w}_{b+1} + \sum_{l=1}^{L} \mathbf{H}_{b+1}^{(\mathrm{R}_l,\mathrm{D})} \mathbf{G}_{b+1}^{(\mathrm{R}_l)} \mathbf{w}_b^{(\mathrm{R}_l)} \end{bmatrix}. \end{aligned} \tag{7}$$

For further simplification of (7) the sum in the compound channel matrix can be expressed in terms of matrix multiplications.

Defining

$$\mathbf{H}_b^{(\mathrm{S,R})} = \left[\mathbf{H}_b^{(\mathrm{S,R}_1)\,T}, \quad \mathbf{H}_b^{(\mathrm{S,R}_2)\,T}, \quad \ldots, \quad \mathbf{H}_b^{(\mathrm{S,R}_L)\,T}\right]^T$$
$$\mathbf{G}_{b+1} = \mathrm{diag}\left[\mathbf{G}_{b+1}^{(\mathrm{R}_1)}, \quad \mathbf{G}_{b+1}^{(\mathrm{R}_2)}, \quad \ldots, \quad \mathbf{G}_{b+1}^{(\mathrm{R}_L)}\right]$$
$$\mathbf{H}_{b+1}^{(\mathrm{R,D})} = \left[\mathbf{H}_{b+1}^{(\mathrm{R}_1,\mathrm{D})}, \quad \mathbf{H}_{b+1}^{(\mathrm{R}_2,\mathrm{D})}, \quad \ldots, \quad \mathbf{H}_{b+1}^{(\mathrm{R}_L,\mathrm{D})}\right]$$
$$\mathbf{w}_b^{(\mathrm{R})} = \left[\mathbf{w}_b^{(\mathrm{R}_1)\,T}, \quad \mathbf{w}_b^{(\mathrm{R}_2)\,T}, \quad \ldots, \quad \mathbf{w}_b^{(\mathrm{R}_L)\,T}\right]^T$$

we obtain

$$\mathbf{r}_c = \begin{bmatrix} \mathbf{H}_b^{(\mathrm{S,D})} & \mathbf{0} \\ \mathbf{H}_{b+1}^{(\mathrm{R,D})}\mathbf{G}_{b+1}\mathbf{H}_b^{(\mathrm{S,R})} & \mathbf{H}_{b+1}^{(\mathrm{S,D})} \end{bmatrix} \begin{bmatrix} \mathbf{s}_b \\ \mathbf{s}_{b+1} \end{bmatrix} + \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{b+1}^{(\mathrm{R,D})}\mathbf{G}_{b+1} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{w}_b \\ \mathbf{w}_b^{(\mathrm{R})} \\ \mathbf{w}_{b+1} \end{bmatrix} \tag{8}$$
$$= \mathbf{H}_c\mathbf{s}_c + \mathbf{K}_c\mathbf{w}_c \tag{9}$$

which is the most general description for our 2-hop cooperative relaying system with linear AF relays.

To derive the mutual information for our system model given in (9) we have to note that the noise in the two time slots has the autocorrelation matrix

$$\mathbf{R}_w = \sigma_{\mathrm{D}}^2 \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \frac{\sigma_{\mathrm{R}}^2}{\sigma_{\mathrm{D}}^2}\mathbf{H}_{b+1}^{(\mathrm{R,D})}\mathbf{G}_{b+1}\mathbf{G}_{b+1}^{\dagger}\mathbf{H}_{b+1}^{(\mathrm{R,D})\,\dagger} + \mathbf{I} \end{bmatrix} = \sigma_{\mathrm{D}}^2\mathbf{Q}_w \tag{10}$$

where σ_{R}^2 and σ_{D}^2 is the variance of the noise at the relays and the destination, respectively.

If we assume that autocorrelation matrix of the transmitted signal is $\mathrm{E}\left\{\mathbf{s}\mathbf{s}^{\dagger}\right\} = P\mathbf{I}$ the mutual information for our system model (9) assuming perfect channel knowledge at the receiver is given by

$$I\left(\mathbf{s}_c; \mathbf{r}_c | \mathbf{H}_c\right) = \frac{1}{2N_{\mathrm{B}}}\log_2\det\left(\mathbf{I} + \frac{P}{\sigma_{\mathrm{D}}^2}\;\mathbf{H}_c\mathbf{H}_c^{\dagger}\mathbf{Q}_w^{-1}\right). \tag{11}$$

The factor $1/2$ reflects the rate loss of the cooperative transmission scheme (2-hop) compared to a direct transmission scheme (1-hop). The normalization factor N_{B}^{-1} is due to the transmitted number of symbols N_{B}.

III. Time Variant Relay Specific Phases

In [1] it is shown that using one AF relay e.g. in traffic pattern T1, second order diversity can be achieved. But the use of more than one AF relay to achieve higher order diversity is not straightforward. If the L relays only retransmit the received signals (without any further signal processing than amplification) the L channel coefficients in (9) add up to only one effective channel coefficient which is constant within one fading interval of length N_{B}. Therefore, there exists no temporal diversity which can be exploited by an outer code. Time-variant and relay specific phase rotations at the relays transform the existing spatial diversity into temporal diversity by creating a time-variant relay fading channel, which can be utilized by an outer code as presented in [9] or [10] to achieve diversity performance. The implementation of these phase rotations is straightforward and leads only to a small extension in signal processing power at the relays.

In the following we describe the time-variant phase rotations within our system model (9) and give some examples. First we have a look on the relay gain matrices, whereas for the sake of simplicity we drop the time indices b and c in our system model in (8) and (9). The gain matrices $\mathbf{G}^{(\mathrm{R}_l)}$ for the L relays can be expressed as a multiplication of two matrices, one amplification matrix $\mathbf{A}^{(\mathrm{R}_l)}$ and one phase signature matrix $\mathbf{P}^{(\mathrm{R}_l)}$

$$\mathbf{G}^{(\mathrm{R}_l)} = \mathbf{A}^{(\mathrm{R}_l)}\mathbf{P}^{(\mathrm{R}_l)} \in \mathbb{C}^{N_{\mathrm{B}}\times N_{\mathrm{B}}}. \tag{12}$$

Both are diagonal matrices, whereas the amplification matrix has time-invariant elements on its diagonal ($\mathbf{A}^{(\mathrm{R}_l)} = a^{(\mathrm{R}_l)}\mathbf{I}$) and the non-zero elements of the phase signature matrix have magnitude one and are therefore power invariant.
The phase signature matrix at relay l is given by

$$\mathbf{P}^{(\mathrm{R}_l)} = \mathrm{diag}\left[p_1^{(\mathrm{R}_l)}, \quad p_2^{(\mathrm{R}_l)}, \quad \ldots, \quad p_{N_{\mathrm{B}}}^{(\mathrm{R}_l)}\right]. \tag{13}$$

We propose for the elements of $\mathbf{P}^{(\mathrm{R}_l)}$ orthogonal phase sequences. One typical sort of orthogonal phase sequences are the columns of a FFT matrix. This leads to

$$p_n^{(\mathrm{R}_l)} = \exp\left(-j\frac{2\pi}{N_{\mathrm{B}}}\left(n-1\right)\left(l-1\right)\right). \tag{14}$$

Another simple choice for $\mathbf{P}^{(\mathrm{R}_l)}$ can be derived from an identity matrix, which leads to elements expressed by

$$p_n^{(\mathrm{R}_l)} = \begin{cases} 1 & \text{if } (n-l+1) \bmod L = 1 \\ 0 & \text{otherwise.} \end{cases} \tag{15}$$

An example for $L = 2$ and $N_{\mathrm{B}} = 4$ is given by

$$\mathbf{P}^{(\mathrm{R}_1)} = \mathrm{diag}\left[1, \quad 0, \quad 1, \quad 0\right]$$
$$\mathbf{P}^{(\mathrm{R}_2)} = \mathrm{diag}\left[0, \quad 1, \quad 0, \quad 1\right].$$

It can be seen that in this case the relays are switched for each symbol which leads to a time-variant channel. One disadvantage of this sequences is that the on-off switching sets high requirements at the linearity of the relays power amplifier.

IV. Impact of the Near-Far Effect

Due to the relative position of the relays to source and destination the choice of $a^{(\mathrm{R}_l)}$ has a crucial impact on the system performance and on the achieved diversity order. A *near-far effect* appears if one relay is far away from the source or in a deep fade and simultaneously near to the destination. In this case the relay mainly amplifies noise, which dominates the resulting SNR at the destination. Therefore a gain allocation scheme which takes this effect into account is necessary.

The capabilities of gain allocation schemes depend on the channel state information (CSI) available at the relay. If the relay has CSI of the source-relay link just as well as of the relay-destination link the gain allocation can try to optimize the SNR at the destination. Note that an overall maximization of the SNR is only possible if all relays jointly optimize their amplification gains.

In this work we only consider CSI of the source-relay link at the relays. This limits the capabilities of the gain allocation scheme. One often quoted scheme determining $a^{(\mathrm{R}_l)}$ is given by

$$a^{(\mathrm{R}_l)} = \alpha\sqrt{\frac{P_{\mathrm{R}}}{P|h^{(\mathrm{S,R}_l)}|^2 + \sigma_{\mathrm{R}}^2}} \tag{16}$$

with $\alpha = 1$ (e.g. in [1]) or $\alpha = h^{(\mathrm{S,R}_l)\,*}/|h^{(\mathrm{S,R}_l)}|$ (e.g. in [8]). P_{R} is the transmit power of one relay. This allocation scheme is very sensitive to deep fades, because in this case this scheme would result in a large amplification gain.

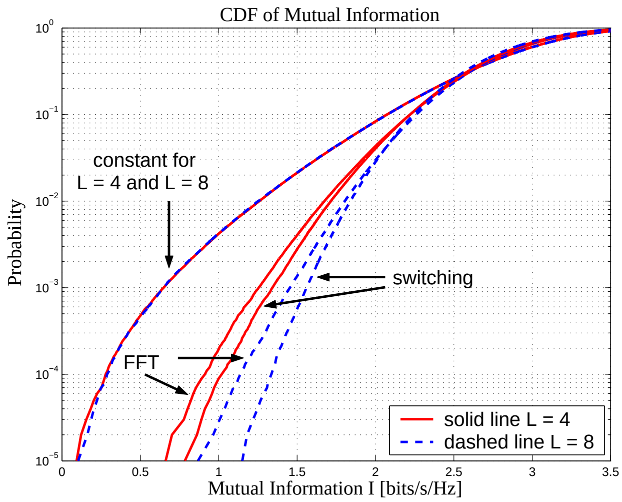

Fig. 3: CDF of mutual information (11) for $L = N_{\mathrm{B}} = 4$ (solid lines) and $L = N_{\mathrm{B}} = 8$ (dashed lines) relays at a SNR = 15dB; constant = relays only amplifies, i.e. $\mathbf{P}^{(\mathrm{R}_l)} = \mathbf{I}$; FFT = orthogonal relay phase signatures (14); switching = relay on-off switching (15)

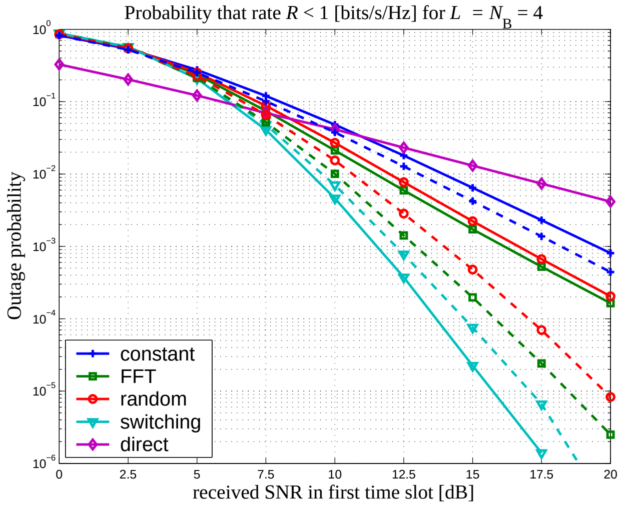

Fig. 4: Outage probability that the rate is less than 1 bit/s/Hz for $L = 4$ relays, blocklength $N_{\mathrm{B}} = 4$ and threshold $a_{\max}$ (dashed lines) and no threshold (solid lines); random = random phase signatures; direct = no relays, source sends in every time slot

To prevent this large gains we propose an extension to (16) by setting an maximal amplification threshold. With this threshold gains are restricted to a specified interval $a^{(\mathrm{R}_l)} \in [0,\ a_{\max}]$. The choice of $a_{\max}$ depends on the number of relays and on the network topology and needs to be optimized [6].

Moreover, we propose a low complexity gain allocation scheme which is based only on the mean received power at the relay. Then $a^{(\mathrm{R}_l)}$ can be expressed by

$$a^{(\mathrm{R}_l)} = \sqrt{\frac{P_{\mathrm{R}}}{P\mathrm{E}\left\{|h^{(\mathrm{S},\mathrm{R}_l)}|^2\right\} + \sigma_{\mathrm{R}}^2}} \tag{17}$$

where the denominator is the mean received power. A further advantage of this allocation scheme is that it is more robust to short deep fades than (16).

V. Performance Results

In this section we will present the performance of our proposed time-variant relay processing by means of computer simulation. We restrict ourself to traffic pattern T1 (Tab. 1).

Simulation Setup: We consider a network where L relays are placed randomly uniform distributed on a disk with radius $r = 750$ wavelength (= 45m at 5Ghz). Source and destination are placed fixed on the border of this disk on opposite sides. We assume channel coefficients which include path loss and small scale-fading:

$$h_k = \frac{1}{d^{\beta/2}} \cdot x_k \tag{18}$$

Here, d is the distance between the two nodes, x_k is a $\mathcal{CN}(0,1)$ complex random variable and $\beta = 2$ is the path loss exponent. Averaging is done over the positions of the relays as over the random variable x_k.

For fairness reasons the relay transmit power P_{R} is chosen such, that the total network power is always P. That means that the power is set to $P_{\mathrm{R}} = P/L$ or $P_{\mathrm{R}} = P$ in the case of (14) or (15), respectively. Note, that in the case of the additional threshold $a_{\max}$ the relays might not use all of their assigned transmit power. Furthermore, we assume equal noise variances at relays and destination, i.e. $\sigma_{\mathrm{D}}^2 = \sigma_{\mathrm{R}}^2$.

Results: In Fig. 3 the cumulative distribution functions of the mutual information (11) for $L = N_{\mathrm{B}} = 4$ (solid lines) and $L = N_{\mathrm{B}} = 8$ (dashed lines) relays are depicted. The received SNR at the destination in the first time slot is 15dB. As gain allocation we choose (16) with $\alpha = 1$ and a suitable threshold $a_{\max}$. A decrease in CDF's variance (steeper slope of CDF at constant mean) stands for an increase in offered diversity. It can be seen that additional relays does not have an effect on the CDF if the relays only retransmit their signals and therefore the channel is time-invariant, i.e. no additional diversity is offered. In comparison the CDFs of the phase rotations (FFT) and the switching always show a smaller variance, which decreases with the number of relays. It can also be seen that the switching method offers a higher diversity than the phase rotation method for both number of relays.

In Fig. 4 the probability that the information rate (11) is less than 1 bit/s/Hz (outage probability) vs. the received SNR at the destination in the first time slot is shown. A steeper slope at the outage probability means a higher order of offered diversity. It can be seen that the performance of cooperative diversity methods is superior (with different performance gain) to the direct transmission between source and destination. To have a fair comparison between cooperative and direct transmission it is assumed that in the direct case the source transmits in both time slots.

First we concentrate on the solid curves, which corresponds to the gain allocation of (16) ($\alpha = 1$) without any additional threshold. It can be seen that only the switching method achieves the full diversity order of $L + 1 = 5$. The random and the FFT phase signatures achieve the same diversity order of two as if the relays only retransmit. In this case only a SNR gain is possible. This shows the importance of our extension to the existing gain allocation of (16). With this additional threshold (dashed curves) the random and the FFT phase signatures method achieve a steeper slope and therefore are able to offer diversity to an appropriate outer code which makes use of the time-variance. Furthermore it can be seen that the performance of the switching method decreases with the use of an amplification threshold, but is still superior to the other meth-

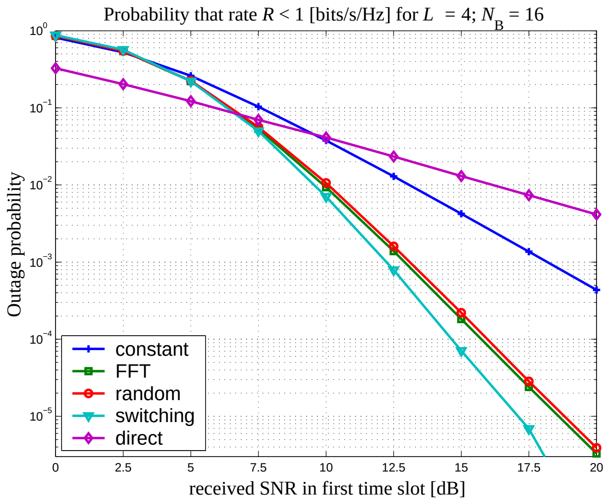

Fig. 5: Outage probability that the rate is less than 1 bit/s/Hz for $L = 4$ relays, blocklength $N_{\mathrm{B}} = 16$ and a threshold $a_{\max}$.

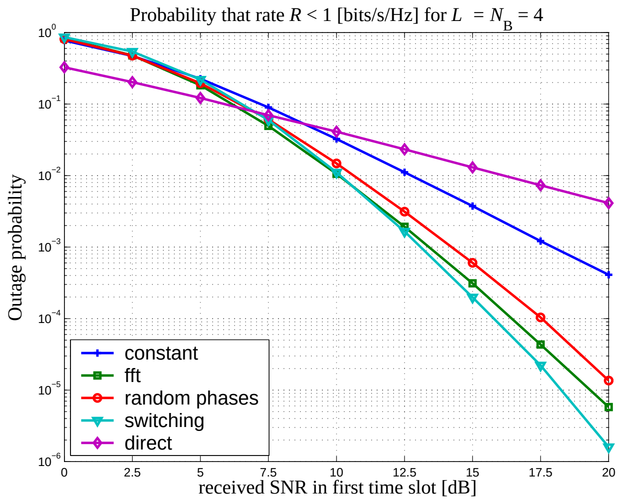

Fig. 6: Outage probability that the rate is less than 1 bit/s/Hz for $L = 4$ relays, blocklength $N_{\mathrm{B}} = 4$ and a gain allocation of (17)

ods. Unfortunately this method sets higher requirements at the linearity of the relays power amplifier.

The importance of a gain allocation scheme which prevents the near-far effect can also be seen in Tab. 2, where the loss in mean SNR in the second slot (difference between mean SNR first slot and mean SNR second slot) is shown. Because of the amplified noise at the relays there is always a loss in SNR. It can

	constant		FFT		switching	
$\mathrm{SNR}_{\mathrm{ref}}$	A	B	A	B	A	B
10 dB	0.26	0.8	0.24	0.78	0.39	0.3
15 dB	0.19	0.83	0.18	0.82	0.33	0.23
20 dB	0.18	0.87	0.18	0.86	0.33	0.22

Tab. 2: Loss in received SNR [dB] in the second time slot compared to first slot; A = gain alloc. of (16) with $a_{\max}$; B = without $a_{\max}$

be seen that the orthogonal phase signatures (FFT) for example show a reduction of the SNR loss if an additional threshold is introduced, which confirms our investigation of the near-far effect. Note, that this reduction appears while the relays do not use their full transmit power in this case. Furthermore it can be seen that for the switching method the loss in received SNR in the second time slot increases with an additional threshold, which explains the decrease in performance for this method.

In Fig. 5 the influence of the blocklength N_{B} is depicted. Compared to Fig. 4 it can be seen that longer orthogonal phase sequences do not improve the diversity performance. Only for the random phase sequences the performance is improved.

In Fig. 6 the performance of the gain allocation of (17) without any further threshold is depicted. This gain allocation scheme has the lowest hardware complexity of the proposed schemes, because the relay only averages over the received power and has not to estimate the actual SNR.
It can be seen that in comparison to Fig. 4 the performance is similar to the gain allocation of (16) with additional threshold. No scheme can offer full diversity of $L + 1 = 5$ but having the low hardware complexity in mind this scheme is quite interesting for implementation e.g. in a sensor network, where all nodes except the destination are sensors (with its specific phase signature), and only the destination as a master node has decoding capabilities.

VI. Conclusions

In this work we proposed time-variant and relay specific phase rotations induced at each relay to make the effective channel time-variant and therefore to achieve diversity. This transformation of spatial diversity into temporal diversity can be used by an appropriate outer code and is available for an arbitrary number of relay nodes. We showed that the allocation of the amplification gains at the relays has great influence on the diversity performance. Furthermore, we gave a low complexity extension to existing gain allocations.

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