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Discrete Event Systems
Exercise 9

1 An Unsolvable Problem

It's the first day of your internship at the software firm Bug Inc., and your boss calls you to his office in order to explain your task for the next three months. He says that many clients complain that the programs of Bug Inc. often contain faulty loops that never terminate. In order to prevent such errors in future, you are asked to implement a program that may check whether a given program will halt on all possible inputs or not.

a) Try to find a proof that convinces your boss that this is not possible for general programs. Hint: The proof works by contradiction. Assume a procedure $halt(P : Program) : boolean$ that takes a program P and decides whether P halts on all possible inputs or not. Now construct a program X that terminates if $halt(X)$ is false and loops endlessly if $halt(X)$ is true, which yields the desired contradiction.

b) Your boss still disagrees and proposes the following method: $halt(X)$ simply simulates the execution of program X. If the program terminates it returns true, and if it loops it returns false. Where is the problem of this approach?

c) Your boss is finally convinced but argues that your proof is a very special case that hardly reflects reality. Are there assumptions under which it is always possible to check whether a program halts or not?

2 Dolce Vita in Rome

In order to relax a little bit from the busy life at ETH, Hector and his girlfriend Rachel decide to spend the weekend in Rome. Besides the cultural attractions, Hector and Rachel are also interested in the great choice of ice cream shops (*gelaterie*) which Rome offers.

During their strolls through Rome, the two students encounter n gelaterie. Assume that these ice cream shops can be ranked uniquely according to their attraction, that is, for any two given shops, Hector and Rachel have a clear preference. For instance, the attraction may be a function of the price of the ice cream, quality, atmosphere of the shop, etc.

Since it's too expensive to eat ice cream on every occasion, the two students apply the following strategy: Whenever a shop i is more attractive than the shops 1 to $i-1$ which they have encountered so far, they buy an ice cream.

Assume that the ice cream shops appear in a random order, i.e., any one of the first i shops is equally likely to be the best so far. How many ice creams do Hector and Rachel consume during the weekend?

3 Soccer Betting

The *FC Basel* soccer club is a particularly moody team. Upon winning a game, they tend to win subsequent games. After losing a game, however, they often end up losing the next game as well. A group of international scientists, consisting of soccer experts, mathematicians, and psychologists, has recently conducted a thorough analysis of this behavior. In particular, they have discovered that upon winning a game, the FCB wins the next game with a probability of 60% as well. With probabilities 0.2 each, the next game will be a tie or a loss. After a loss, the FCB will win/tie/lose its next game with probability 0.1/0.2/0.7, respectively. Finally, after a tie, the next game being a win or a loss is equally probable. The probability that the next game also ends up being a tie is 0.4.

a) Model the FCB's moodiness using a Markov chain.

b) In *three games* from now, the FCB will play against the Grasshoppers from Zurich. The Swiss TOTO offers your the following odds:

   Win: 3.5 Tie: 3.5 Loss: 2.0

   Given that the FCB won two games ago, but lost its last game, what would be your bet? Why?

c) More recent studies have shown that the FCB is even moodier than expected. In fact, after losing *two games in a row*, the probability of winning its next game reduces to 0.05, that of getting at least a tie to 0.1. Change your Markov chain model to incorporate the new circumstances. How does the change influence your bet?