Discrete Event Systems
Exercise 11

1 "Hopp FCB!"

Besides its moodiness, the *FC Basel* (FCB) soccer club is confronted with yet another problem: sporadically, its players get sick. Assume that the whole team consists of n players. Assume that the time period a fit player remains fit is exponentially distributed with parameter μ (independently of the state of the other players). On the other hand, the time a sick player remains sick is exponentially distributed with parameter λ.

a) Model the situation as a birth-and-death Markov process where the states denote the number of players which are fit.

b) Derive a formula for the probability that exactly i players are fit.

c) Assume that the FCB has 20 players, and that $\lambda^{-1} = 4$ months and $\mu^{-1} = 10$ months. Calculate the probability that the FCB can participate at a given match.

2 A Binary Game

Anna and Markus play the following game: Well hidden from the other player, they write either 0 or 1 onto a note. Then, they disclose their decision. If the sum modulo 2 is 0, Anna wins, and vice versa if the sum is 1.

a) Anna's strategy is to write both 0 and 1 with probability .5, independently of the past. Markus on the other hand writes 0 and 1 with probabilities .4 and .6 respectively. Who will win more games on average?

b) Assume that Anna changes her strategy as follows: She writes the number with which she would have won the last game, i.e., if Markus has written 0 in the last round, Anna writes 0, and if Markus has written 1, Anna writes 1. Assuming Markus' strategy is unchanged, who will win more games now (on average)?

c) Finally, Markus changes his strategy as well: While Anna writes the number with which she would have won the last game, Markus writes the number with which he would have won *two* games ago. Analyze these strategies as well!

3 Gloriabar

Every day, 540 students, professors and other personnel of ETH go to the Gloriabar for lunch between 11:45h and 13:15h. There is only one queue, and the cashier needs on average 9secs to serve a person. Assume that the arrival and service times are exponentially distributed. Moreover, assume that we do not model the process in which a customer gets its food.

a) Compute the expected waiting time until a student reaches the cashier and the expected waiting time until she has paid for the food.

b) Compute the length of the queue (without the person who is being served).

c) What should be the service time such that the waiting time until a student gets her menu is cut in half?

4 Queuing Networks

Customers of the Internet Service Provider *RedWindow* who have problems with their Internet access, can call a hot-line. There, a customer must first talk to a dispatcher. The dispatcher is very moody and with probability p_d, he kicks people out of the line. However, with probability $1 - p_d$, a customer is connected to a technician. The technician can solve the problem with probability p_t. However, if he can not solve it, he claims that it's the fault of the monopolistic modem producer Beep. Thus, with probability $1 - p_t$, the customer has to call *Beep*. Unfortunately, the agent at Beep can solve the problem only with probability p_b. With probability $1 - p_b$, the customer is told that it's indeed the fault of RedWindow, and hence the customer is connected back to the dispatcher of RedWindow. Etc.!

In the following, we assume that a customer which calls RedWindow for the second time experiences exactly the same success probabilities as in the first round. Let now λ be the Poisson distributed arrival rate (per hour) of the *direct* (i.e., not reconnected) calls to RedWindow. Moreover, assume that the technician of RedWindow and the agent of Beep do not get additional (direct) calls. The service times of the dispatcher, the technician and the agent are exponentially distributed with parameter μ_d (dispatcher), μ_t (technician) and μ_b (Beep agent). If the dispatcher, the technician or the agent are occupied, the customer is put into the waiting line of the corresponding person.

a) Model the situation using the techniques from the lecture.

b) Describe the arrival rate of the phone calls at the technician of RedWindow as a function of p_d, p_t, p_b and λ!

c) How long is a customer after he has been forwarded from the dispatcher in the waiting queue of the technician until he is served (on average)?

d) Now assume that $p_d = 1/6$, $p_t = 1/5$, $p_b = 1/4$, and $\lambda = 5$ per hour. Moreover, let $\mu_d = 20$ per hour, $\mu_t = 10$ per hour, and $\mu_b = 10$ per hour. Compute the expected number of customers in the system (of both RedWindow and Beep together)! What is the expected time a customer is in the system?

e) The technician of RedWindow is quite lazy and hence tells the dispatcher to kick customers out of the line with such a high probability that he only gets one call per hour. Compute the new p_d assuming that all other parameters stay the same!

5 Theory of Ice Cream Vending

Olga and Pascal sell ice cream. In order to serve one customer, each of them needs an amount of time which is exponentially distributed with parameter μ. There is one line of customers in front of their shop, and new customers arrive with a rate λ. Under which conditions is there an equilibrium for this system? And what is the probability that there is no customer in the system (in the steady state)?