



Geo-Routing

Chapter 3

Rating

- Area maturity

First steps

Text book

- Practical importance

No apps

Mission critical

- Theoretical importance

Not really

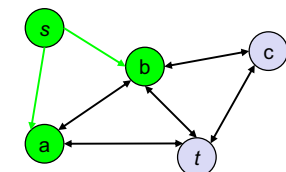
Must have

Overview

- Classic routing overview
- Geo-routing
- Greedy geo-routing
- Euclidean and planar graphs
- Face Routing
- Greedy and Face Routing
- 3D Geo-Routing
- Geometric Routing without Geometry

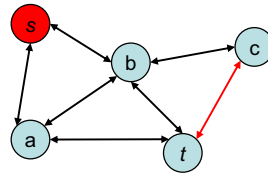
Classic Routing 1: Flooding

- What is Routing?
- „Routing is the act of moving information across a network from a source to a destination.“ (CISCO)
- The simplest form of routing is “flooding”: a source s sends the message to all its neighbors; when a node other than destination t receives the message the first time it re-sends it to all its neighbors.
- + simple (sequence numbers)
- a node might see the same message more than once. (How often?)
- what if the network is huge but the target t sits just next to the source s ?
- We need a smarter routing algorithm



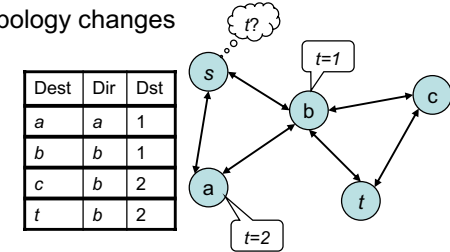
Classic Routing 2: Link-State Routing Protocols

- Link-state routing protocols are a preferred iBGP method (within an autonomous system) in the Internet
- Idea: periodic notification of all nodes about the complete graph
- Routers then forward a message along (for example) the shortest path in the graph
- + message follows shortest path
- every node needs to store whole graph, even links that are not on any path
- every node needs to send and receive messages that describe the whole graph regularly



Classic Routing 3: Distance Vector Routing Protocols

- The predominant method for wired networks
- Idea: each node stores a routing table that has an entry to each destination (destination, distance, neighbor)
- If a router notices a change in its neighborhood or receives an update message from a neighbor, it updates its routing table accordingly and sends an update to all its neighbors
- + message follows shortest path
- + only send updates when topology changes
- most topology changes are irrelevant for a given source/destination pair
- every node needs to store a big table
- count-to-infinity problem



Discussion of Classic Routing Protocols

- **Proactive** Routing Protocols
- **Reactive** Routing Protocols
- Both link-state and distance vector are "proactive," that is, routes are established and updated even if they are never needed.
- Flooding is "reactive," but does not scale
- If there is **almost no mobility**, proactive algorithms are superior because they never have to exchange information and find optimal routes easily.
- If **mobility is high** and data transmission rare, reactive algorithms are superior; in the extreme case of almost no data and very much mobility the simple flooding protocol might be a good choice.

There is **no "optimal" routing protocol**; the choice of the routing protocol depends on the circumstances. Of particular importance is the mobility/data ratio.



Routing in Ad-Hoc Networks

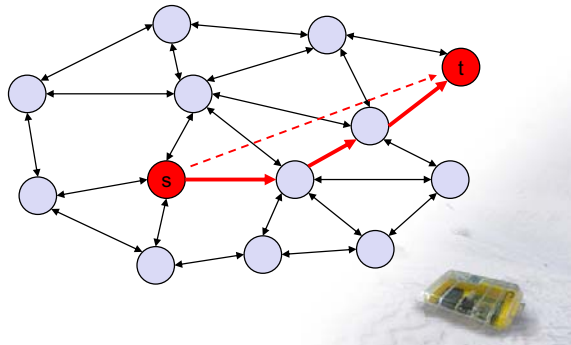
- **Reliability**
 - Nodes in an ad-hoc network are not 100% reliable
 - Algorithms need to find alternate routes when nodes are failing
- **Mobile Ad-Hoc Network (MANET)**
 - It is often assumed that the nodes are mobile ("Moteran")
- 10 Tricks → 2¹⁰ routing algorithms
- In reality there are almost that many proposals!
- Q: How good are these routing algorithms?!? **Any hard results?**
- A: Almost none! Method-of-choice is simulation...
- "If you simulate three times, you get three different results"



Geometric (geographic, directional, position-based) routing

- ...even with all the tricks there will be flooding every now and then.
- In this chapter we will assume that the nodes are location aware (they have GPS, Galileo, or an ad-hoc way to figure out their coordinates), and that we know where the destination is.

- Then we simply route towards the destination

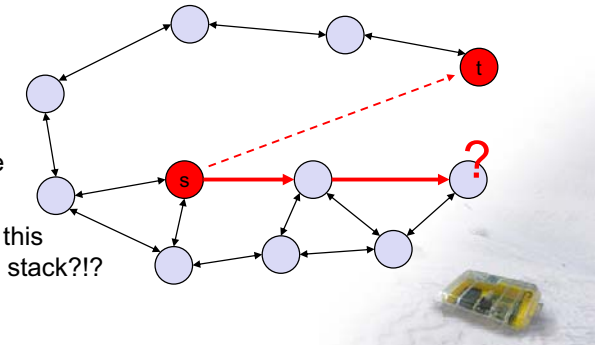


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Geometric routing

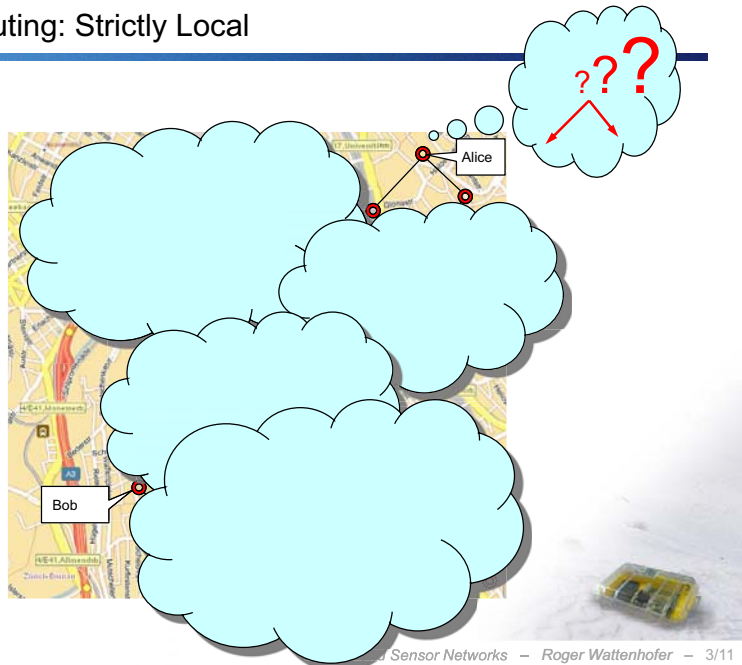
- Problem: What if there is no path in the right direction?
- We need a guaranteed way to reach a destination even in the case when there is no directional path...
- Hack: as in flooding nodes keep track of the messages they have already seen, and then they backtrack* from there

*backtracking? Does this mean that we need a stack?!?



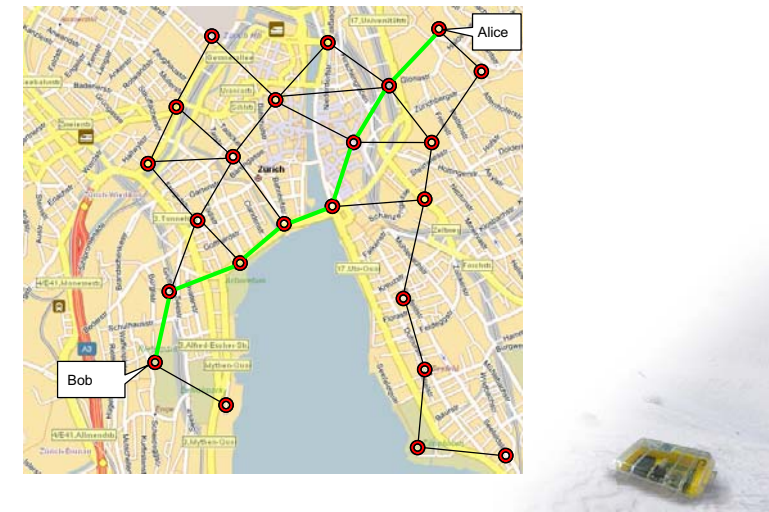
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Geo-Routing: Strictly Local



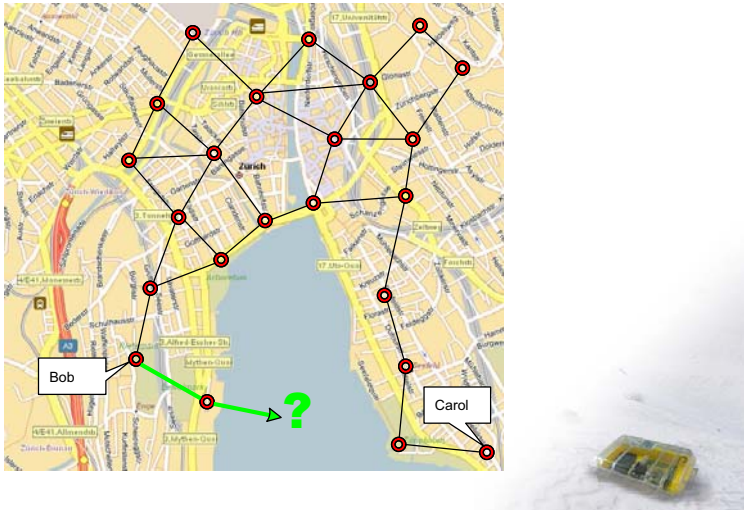
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Greedy Geo-Routing?



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Greedy Geo-Routing?



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What is Geographic Routing?

- A.k.a. geometric, location-based, position-based, etc.

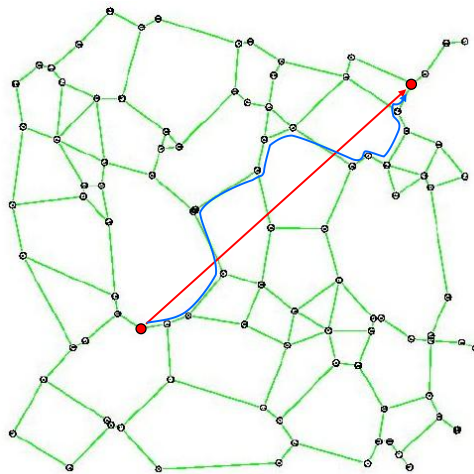
- Each node knows its own position and position of neighbors
- Source knows the position of the destination
- **No routing tables stored in nodes!**

- Geographic routing makes sense
 - Own position: GPS/Galileo, local positioning algorithms
 - Destination: Geocasting, location services, source routing++
 - **Learn about ad-hoc routing in general**

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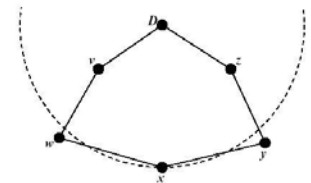
Greedy routing

- Greedy routing looks promising.
- Maybe there is a way to choose the next neighbor and a particular graph where we always reach the destination?

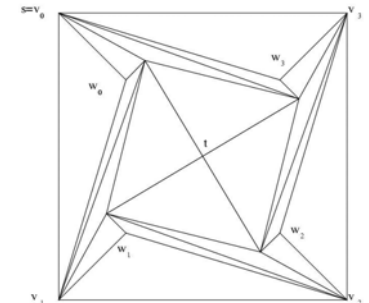


Examples why greedy algorithms fail

- We greedily route to the neighbor which is closest to the destination: But both neighbors of x are not closer to destination D

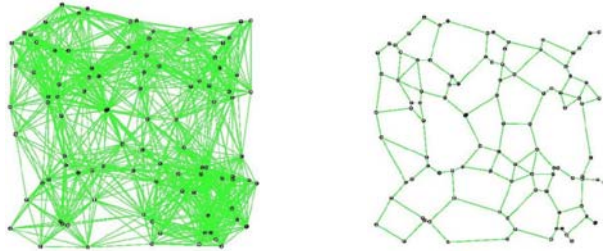


- Also the best angle approach might fail, even in a triangulation: if, in the example on the right, you always follow the edge with the narrowest angle to destination t , you will forward on a loop $v_0, w_0, v_1, w_1, \dots, v_3, w_3, v_0, \dots$



Euclidean and Planar Graphs

- Euclidean: Points in the plane, with coordinates
- Planar: can be drawn without “edge crossings” in a plane

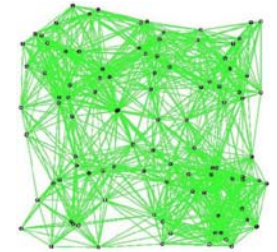


- Euclidean planar graphs (planar embeddings) simplify geometric routing.

Unit disk graph

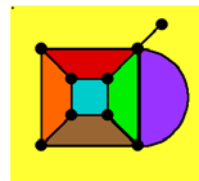
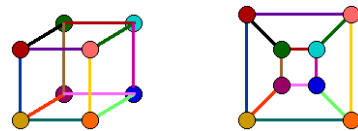
- We are given a set V of nodes in the plane (points with coordinates).
- The unit disk graph $UDG(V)$ is defined as an undirected graph (with E being a set of undirected edges). There is an edge between two nodes u, v iff the Euclidean distance between u and v is at most 1.
- Think of the unit distance as the maximum transmission range.

- We assume that the unit disk graph UDG is connected (that is, there is a path between each pair of nodes)
- The unit disk graph has many edges.
- Can we drop some edges in the UDG to reduce complexity?



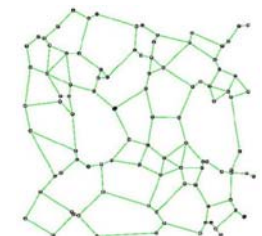
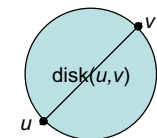
Planar graphs

- Definition: A planar graph is a graph that can be drawn in the plane such that its edges only intersect at their common end-vertices.
- Kuratowski's Theorem: A graph is planar iff it contains no subgraph that is edge contractible to K_5 or $K_{3,3}$.
- Euler's Polyhedron Formula: A connected planar graph with n nodes, m edges, and f faces has $n - m + f = 2$.
- Right: Example with 9 vertices, 14 edges, and 7 faces (the yellow “outside” face is called the infinite face)
- Theorem: A simple planar graph with n nodes has at most $3n - 6$ edges, for $n \geq 3$.



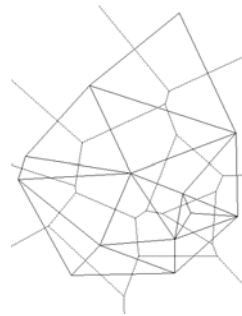
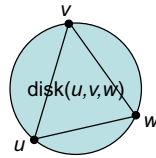
Gabriel Graph

- Let $\text{disk}(u, v)$ be a disk with diameter (u, v) that is determined by the two points u, v .
- The Gabriel Graph $GG(V)$ is defined as an undirected graph (with E being a set of undirected edges). There is an edge between two nodes u, v iff the $\text{disk}(u, v)$ including boundary contains no other points.
- As we will see the Gabriel Graph has interesting properties.



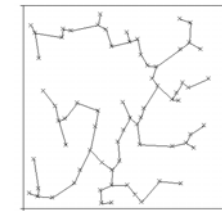
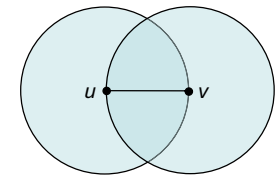
Delaunay Triangulation

- Let $\text{disk}(u,v,w)$ be a disk defined by the three points u,v,w .
- The Delaunay Triangulation (Graph) $\text{DT}(V)$ is defined as an undirected graph (with E being a set of undirected edges). There is a triangle of edges between three nodes u,v,w iff the $\text{disk}(u,v,w)$ contains no other points.
- The Delaunay Triangulation is the dual of the Voronoi diagram, and widely used in various CS areas; the DT is planar; the distance of a path (s,\dots,t) on the DT is within a constant factor of the s - t distance.



Other planar graphs

- Relative Neighborhood Graph $\text{RNG}(V)$
- An edge $e = (u,v)$ is in the $\text{RNG}(V)$ iff there is no node w with $(u,w) < (u,v)$ and $(v,w) < (u,v)$.
- Minimum Spanning Tree $\text{MST}(V)$
- A subset of E of G of minimum weight which forms a tree on V .

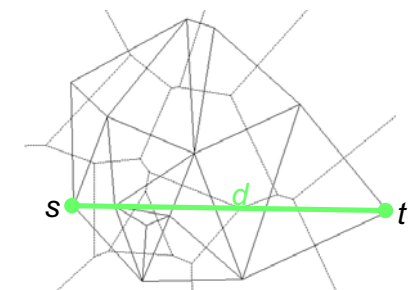


Properties of planar graphs

- Theorem 1:
 $\text{MST}(V) \subseteq \text{RNG}(V) \subseteq \text{GG}(V) \subseteq \text{DT}(V)$
- Corollary:
Since the $\text{MST}(V)$ is connected and the $\text{DT}(V)$ is planar, all the planar graphs in Theorem 1 are connected and planar.
- Theorem 2:
The Gabriel Graph contains the Minimum Energy Path (for any path loss exponent $\alpha \geq 2$)
- Corollary:
 $\text{GG}(V) \cap \text{UDG}(V)$ contains the Minimum Energy Path in $\text{UDG}(V)$

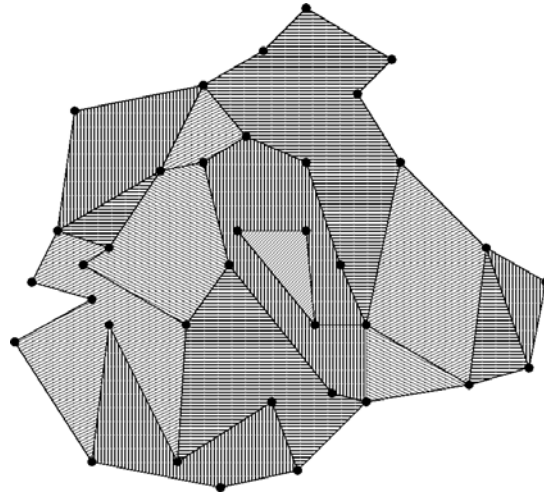
Routing on Delaunay Triangulation?

- Let d be the Euclidean distance of source s and destination t
- Let c be the sum of the distances of the links of the shortest path in the Delaunay Triangulation
- It was shown that $c = \Theta(d)$
- Three problems:
 - 1) How do we find this best route in the DT? With flooding?!?
 - 2) How do we find the DT at all in a distributed fashion?
 - 3) Worse: The DT contains edges that are not in the UDG, that is, nodes that cannot receive each other are "neighbors" in the DT



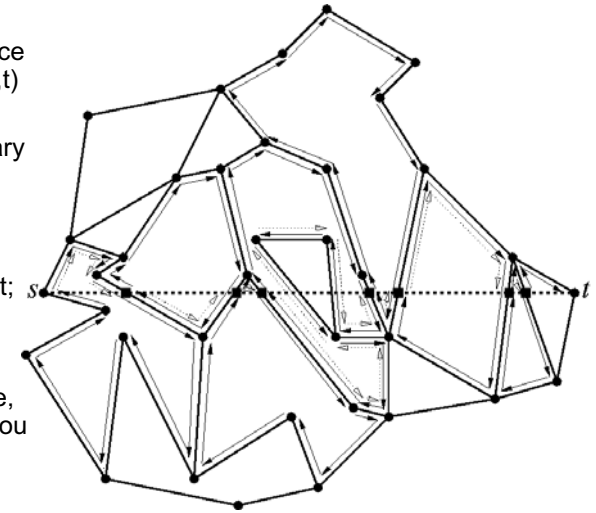
Breakthrough idea: route on faces

- Remember the faces...
- Idea: Route along the boundaries of the faces that lie on the source-destination line

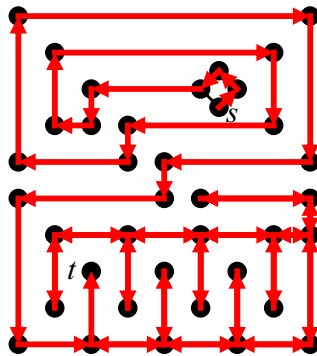


Face Routing

- Let f be the face incident to the source s , intersected by (s,t)
- Explore the boundary of f ; remember the point p where the boundary intersects with (s,t) which is nearest to t ; after traversing the whole boundary, go back to p , switch the face, and repeat 1 until you hit destination t .

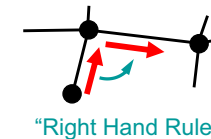
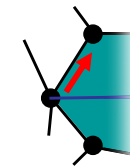


Face Routing Works on Any Graph



Face Routing Properties

- All necessary information is stored in the message
 - Source and destination positions
 - Point of transition to next face
- Completely local:
 - Knowledge about direct neighbors' positions sufficient
 - Faces are **implicit**



"Right Hand Rule"

- Planarity** of graph is **computed** locally (not an assumption)
 - Computation for instance with Gabriel Graph



Face routing is correct

- Theorem: Face routing terminates on any simple planar graph in $O(n)$ steps, where n is the number of nodes in the network
- Proof: A simple planar graph has at most $3n-6$ edges. You leave each face at the point that is closest to the destination, that is, you never visit a face twice, because you can order the faces that intersect the source—destination line on the exit point. Each edge is in at most 2 faces. Therefore each edge is visited at most 4 times. The algorithm terminates in $O(n)$ steps.



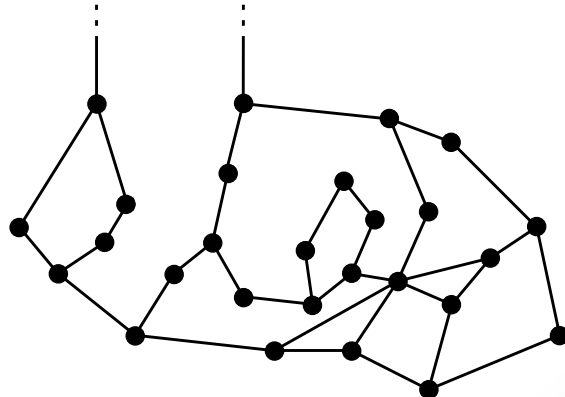
Is there something better than Face Routing?

- How to improve face routing? A proposal called “Face Routing 2”
- Idea: Don't search a whole face for the best exit point, but take the first (better) exit point you find. Then you don't have to traverse huge faces that point away from the destination.
- Efficiency: Seems to be practically more efficient than face routing. But the theoretical worst case is worse – $O(n^2)$.
- Problem: if source and destination are very close, we don't want to route through all nodes of the network. Instead we want a routing algorithm where the cost is a function of the cost of the best route in the unit disk graph (and independent of the number of nodes).

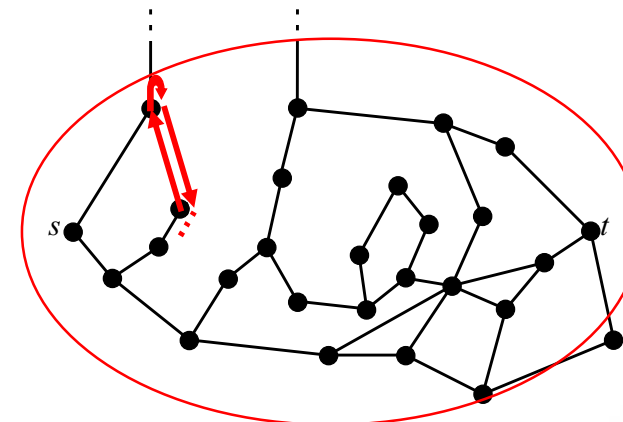


Face Routing

- Theorem: Face Routing reaches destination in $O(n)$ steps
- But: Can be very bad compared to the optimal route

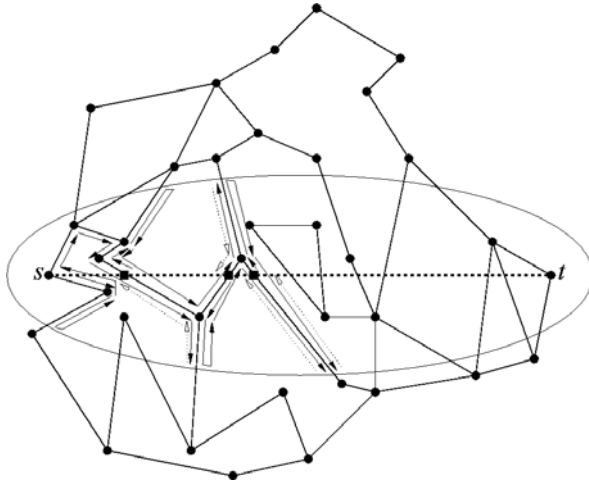


Bounding Searchable Area



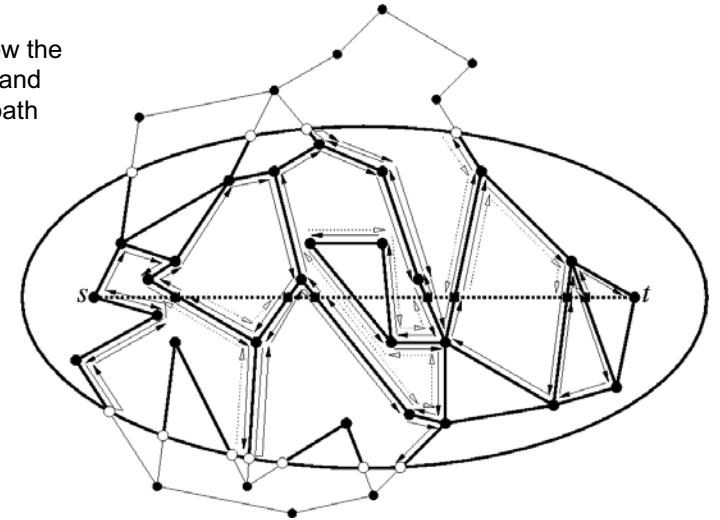
Adaptive Face Routing (AFR)

- Idea: Use face routing together with ad hoc routing trick 1!!
- That is, don't route beyond some radius r by branching the planar graph within an ellipse of exponentially growing size.



AFR Example Continued

- We grow the ellipse and find a path



AFR Pseudo-Code

0. Calculate $G = GG(V) \cap UDG(V)$
Set c to be twice the Euclidean source—destination distance.
 1. Nodes $w \in W$ are nodes where the path $s-w-t$ is larger than c . Do face routing on the graph G , but without visiting nodes in W . (This is like pruning the graph G with an ellipse.) You either reach the destination, or you are stuck at a face (that is, you do not find a better exit point.)
 2. If step 1 did not succeed, double c and go back to step 1.
- Note: All the steps can be done completely locally, and the nodes need no local storage.



The $\Omega(1)$ Model

- We simplify the model by assuming that nodes are sufficiently far apart; that is, there is a constant d_0 such that all pairs of nodes have at least distance d_0 . We call this the $\Omega(1)$ model.
- This simplification is natural because nodes with transmission range 1 (the unit disk graph) will usually not “sit right on top of each other”.
- Lemma: In the $\Omega(1)$ model, all natural cost models (such as the Euclidean distance, the energy metric, the link distance, or hybrids of these) are equal up to a constant factor.
- Remark: The properties we use from the $\Omega(1)$ model can also be established with a backbone graph construction.



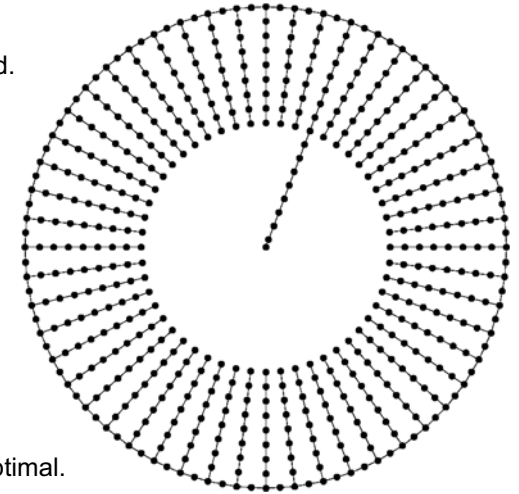
Analysis of AFR in the $\Omega(1)$ model

- Lemma 1: In an ellipse of size c there are at most $O(c^2)$ nodes.
- Lemma 2: In an ellipse of size c , face routing terminates in $O(c^2)$ steps, either by finding the destination, or by not finding a new face.
- Lemma 3: Let the optimal source—destination route in the UDG have cost c^* . Then this route c^* must be in any ellipse of size c^* or larger.
- Theorem: AFR terminates with cost $O(c^{*2})$.
- Proof: Summing up all the costs until we have the right ellipse size is bounded by the size of the cost of the right ellipse size.



Lower Bound

- The network on the right constructs a lower bound.
- The destination is the center of the circle, the source any node on the ring.
- Finding the right chain costs $\Omega(c^{*2})$, even for randomized algorithms
- Theorem: AFR is asymptotically optimal.



Non-geometric routing algorithms

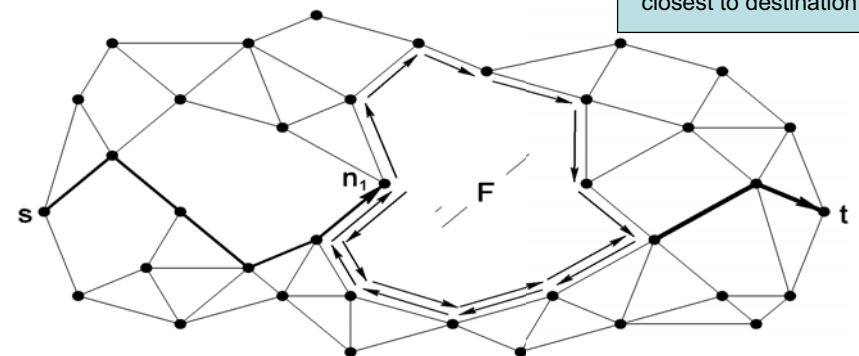
- In the $\Omega(1)$ model, a standard flooding algorithm enhanced with trick 1 will (for the same reasons) also cost $O(c^{*2})$.
- However, such a flooding algorithm needs $O(1)$ extra storage at each node (a node needs to know whether it has already forwarded a message).
- Therefore, there is a trade-off between $O(1)$ storage at each node or that nodes are location aware, and also location aware about the destination. This is intriguing.



GOAFR – Greedy Other Adaptive Face Routing

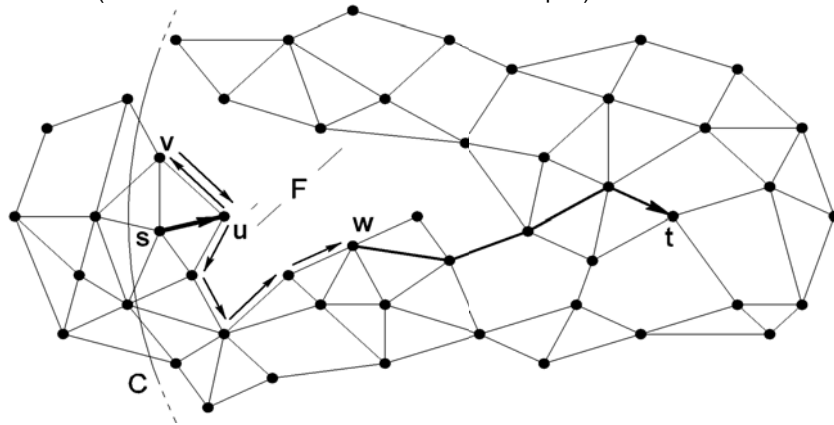
- Back to geometric routing...
- AFR Algorithm is not very efficient (especially in dense graphs)
- Combine Greedy and (Other Adaptive) Face Routing
 - Route greedily as long as possible
 - Circumvent “dead ends” by use of face routing
 - Then route greedily again

Other AFR: In each face proceed to node closest to destination



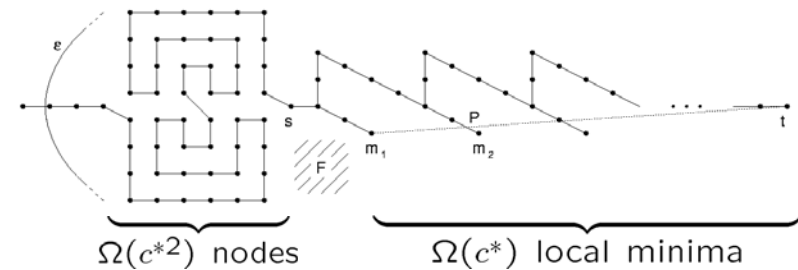
GOAFR+

- GOAFR+ improvements:
 - Early fallback to greedy routing
 - (Circle centered at destination instead of ellipse)



Early Fallback to Greedy Routing?

- We could fall back to greedy routing as soon as we are closer to t than the local minimum
- But:



- “Maze” with $\Omega(c^*2)$ edges is traversed $\Omega(c^*)$ times $\rightarrow \Omega(c^*3)$ steps

GOAFR – Greedy Other Adaptive Face Routing

- Early fallback to greedy routing:
 - Use counters p and q . Let u be the node where the exploration of the current face F started
 - p counts the nodes closer to t than u
 - q counts the nodes *not* closer to t than u
 - Fall back to greedy routing as soon as $p > \sigma \cdot q$ (constant $\sigma > 0$)

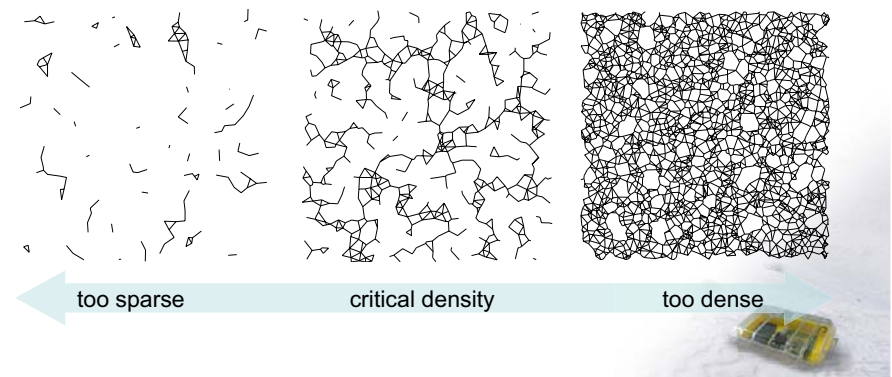
Theorem: GOAFR is still asymptotically worst-case optimal...
...and it is efficient in practice, in the average-case.

- What does “practice” mean?
 - Usually nodes placed uniformly at random



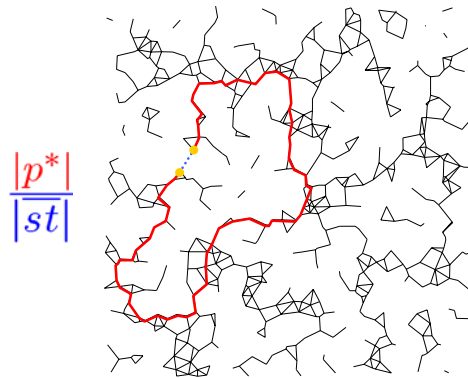
Average Case

- Not interesting when graph not dense enough
- Not interesting when graph is too dense
- **Critical density range** (“percolation”)
 - Shortest path is significantly longer than Euclidean distance



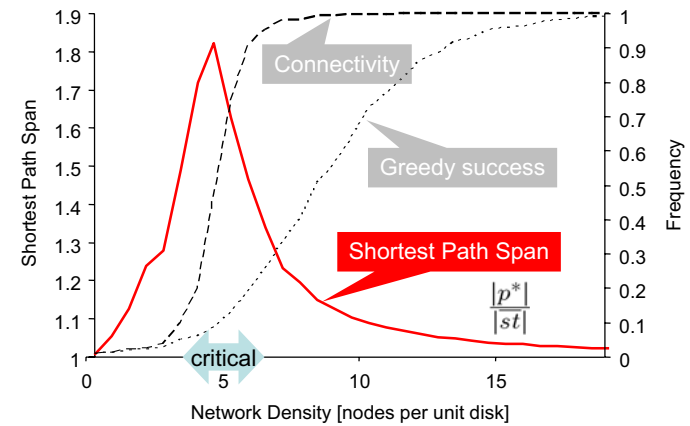
Critical Density: Shortest Path vs. Euclidean Distance

- Shortest path is significantly longer than Euclidean distance

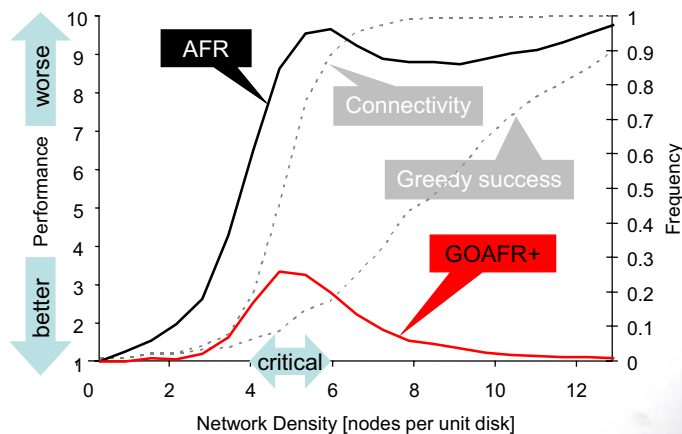


- Critical density range mandatory for the simulation of **any** routing algorithm (not only geographic)

Randomly Generated Graphs: Critical Density Range



Simulation on Randomly Generated Graphs



A Word on Performance

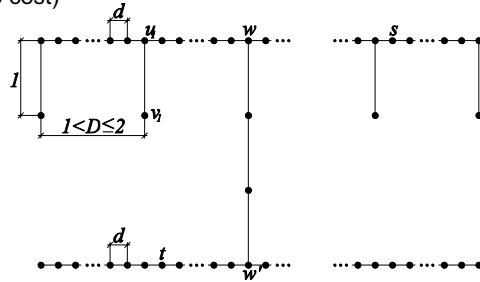
- What does a performance of 3.3 in the critical density range mean?
- If an **optimal path** (found by Dijkstra) has **cost c**, then **GOAFR+** finds the destination **in 3.3·c steps**.
- It does *not* mean that the *path* found is 3.3 times as long as the optimal path! The path found can be much smaller...
- Remarks about cost metrics
 - In this lecture “cost” $c = c$ hops
 - There are other results, for instance on distance/energy/hybrid metrics
 - In particular: With energy metric there is no competitive geometric routing algorithm

Energy Metric Lower Bound

Example graph: k "stalks", of which only one leads to t

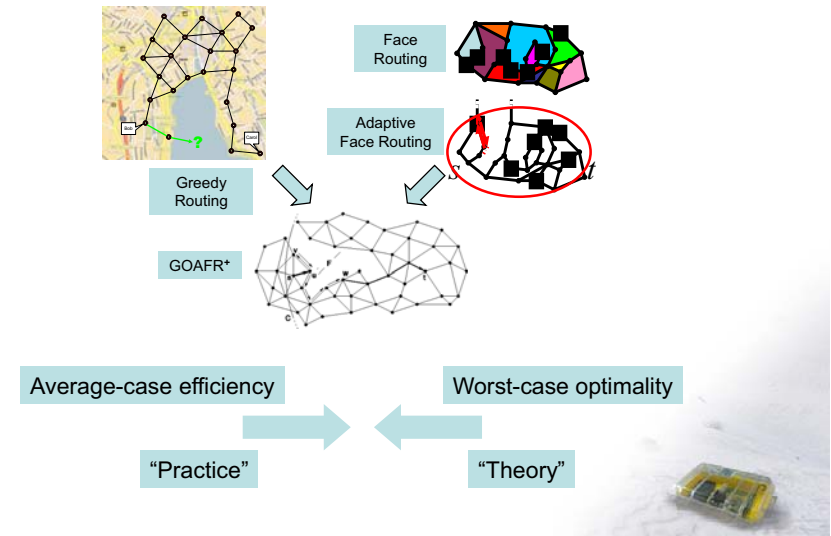
- any deterministic (randomized) geometric routing algorithm A has to visit all k (at least $k/2$) "stalks"
- optimal path has constant cost c^* (covering a constant distance at almost no cost)

$$\lim_{k \rightarrow \infty} \frac{c(A)}{c^*} = \infty$$



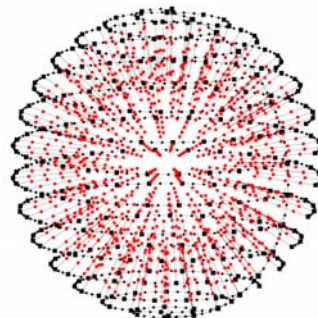
→ With energy metric there is no competitive geometric routing algorithm

GOAFR: Summary



3D Geo-Routing

- The world is not flat. We can certainly envision networks in 3D, e.g. in a large office building. Can we geo-route in three dimensions? Are the same techniques possible?
- Certainly, if the node density is high enough (and the node distribution is kind to us), we can simply use greedy routing. But what about those local minima?!?
- Is there something like a face in 3D?
- **How would you do 3D routing?**
- The picture on the right is the 3D equivalent of the 2D lower bound, proving that we need at least OPT^3 steps.



Deterministic Routing in 3-Dimensional Networks

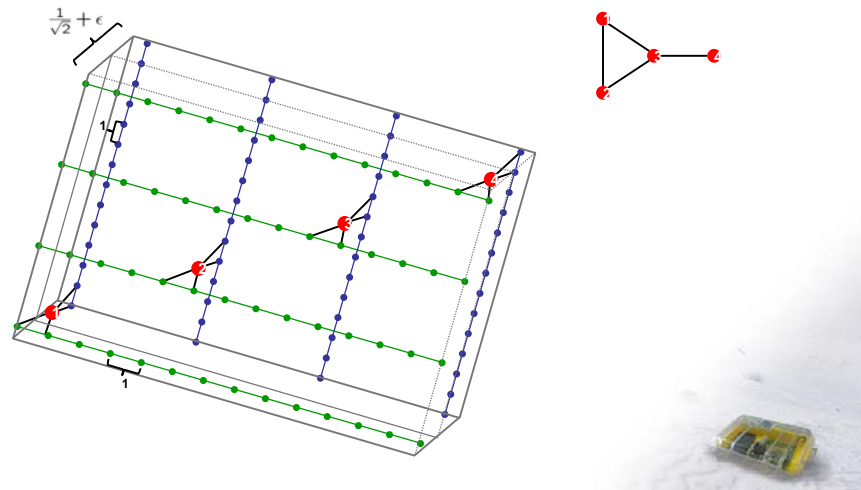
We will prove that

There is no deterministic k -local routing algorithm for 3D UDGs

- *Deterministic*: Whenever a node n receives a message from node m , n determines the next hop as a function $f(n, m, s, t, N(n))$, where s and t are the source and the target nodes and $N(n)$ the neighborhood of n .
- *k -local*: A node only knows its k -hop neighborhood
- **Proof Outline**:
 - We show that an arbitrary graph G can be translated to a 3D UDG G'
 - Assume for contradiction that there is a k -local algorithm A_k for 3D UDGs,
 - We show that there must also be a 1-local algorithm A_1 for 3D UDGs
 - The translation from G to G' is strictly local, therefore, we could simulate A_1 on G and obtain a 1-local routing for arbitrary graphs
 - We show that there is no such algorithm, disproving the existence of A_k .

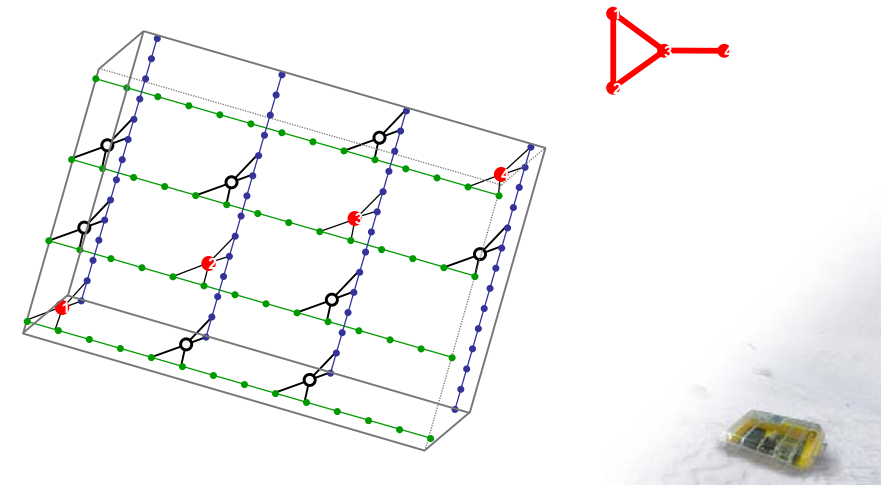
Transforming a general graph to a 3D UDG (1/2)

- Main idea: Build the 3D UDG similar to an electronic circuit on three layers, and add chains of virtual nodes (the conductors)



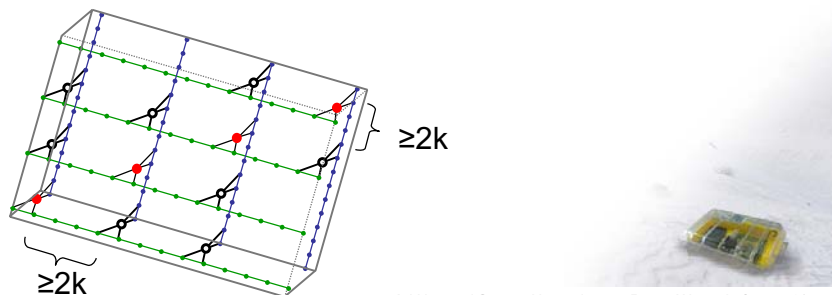
Transforming a general graph to a 3D UDG (2/2)

- Virtual nodes on the middle layer establish the connections
- The resulting graph is a 3D UDG



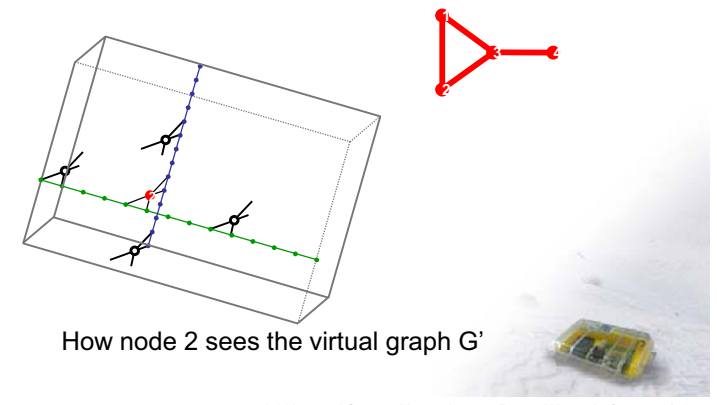
1-local Routing for 3D UDGs

- Assume that there is a k -local routing algorithm A_k for 3D UDG
- Adapt the transformation s.t. the connecting lines contain at least $2k$ virtual nodes
- As a result, A_k cannot see more than 1 hop of the original graph
- The stretching of the paths introduces 'dummy' information of no use, but the algorithm A_k still has to work
- Therefore, there must also be a 1-local algorithm A_1 for 3D UDG



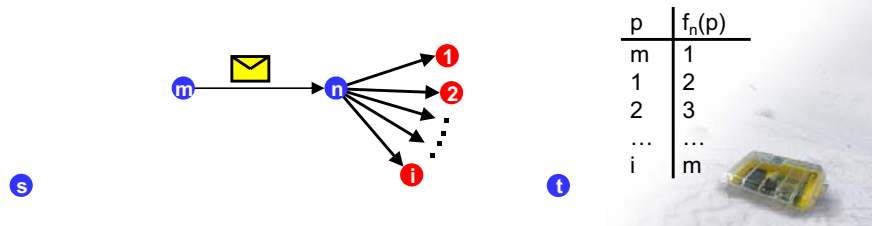
1-local Routing for Arbitrary Graphs

- The transformation to the 3D UDG G' can be determined strictly locally from any graph G
- The nodes of any graph G can simulate A_1 by simulating G'
- Therefore, A_1 can be used to build a 1-local routing algorithm for arbitrary graphs



1-local Routing for Arbitrary Graphs is *impossible* (1/2)

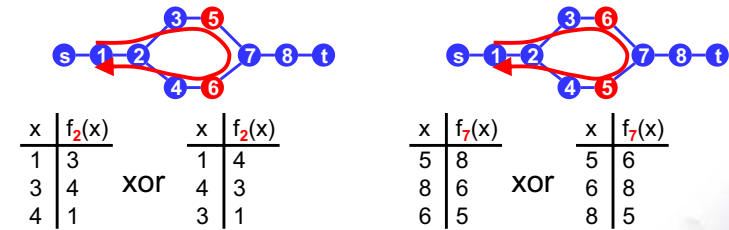
- A *deterministic* routing algorithm can be described as a function $f(n,m,s,t,N(n))$, which returns the next hop
- n : current node, m : previous node, s : source, t : target, $N(n)$: neighborhood of n
- Node n has no means to determine *locally* which of its neighbors has a connection to $t \rightarrow n$ must try *all* of them before returning to m
- Even the position of t or s can't help
- The function f must be a cycle over the $i+1$ neighbors
- If not, we miss some neighbors of n , which may connect to t



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1-local Routing for Arbitrary Graphs is *impossible* (2/2)

- Node 2 and 7 have to decide on one forwarding function
- There are 4 combinations possible. For all of them, forwarding fails either in the left or the right network
- Conclusion 1: 1-local routing algorithms do not exist
- Conclusion 2: There is no k-local routing algorithm for 3D UDG
- Conclusion 3: There is no k-local routing algorithm for 3D graphs



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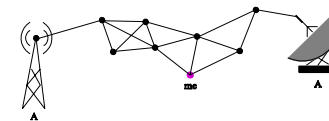
Routing with and without position information

- **Without** position information:
 - Flooding
→ **does not scale**
 - Distance Vector Routing
→ **does not scale**
 - Source Routing
 - **increased per-packet overhead**
 - **no theoretical results, only simulation**
- **With** position information:
 - Greedy Routing
→ **may fail**: message may get stuck in a “dead end”
 - Geometric Routing
→ It is assumed that each node **knows its position**

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Obtaining Position Information

- Attach GPS to each sensor node
 - Often undesirable or impossible
 - GPS receivers clumsy, expensive, and energy-inefficient
- Equip only a few designated nodes with a GPS
 - **Anchor** (landmark) nodes have GPS
 - Non-anchors derive their position through communication (e.g., count number of hops to different anchors)

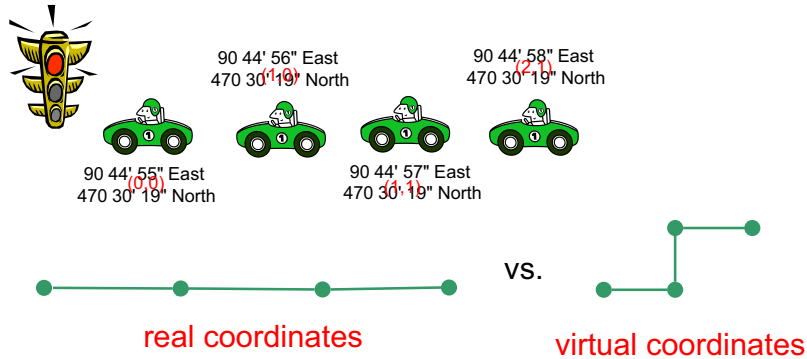


Anchor density determines quality of solution

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What about no GPS at all?

- In absence of GPS-equipped anchors...
 - ...nodes are clueless about **real coordinates**.
- For many applications, real coordinates are not necessary
 - **Virtual coordinates** are sufficient



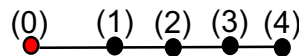
What are „good“ virtual coordinates?

- Given the **connectivity information** for each node and knowing the underlying graph is a UDG find **virtual coordinates** in the plane such that all connectivity requirements are fulfilled, i.e. find a **realization (embedding)** of a UDG:
 - each edge has length at most 1
 - between non-neighbored nodes the distance is more than 1
- Finding a **realization** of a UDG from **connectivity information** only is NP-hard...
 - [Breu, Kirkpatrick, Comp.Geom.Theory 1998]
- ...and also hard to **approximate**
 - [Kuhn, Moscibroda, Wattenhofer, DIALM 2004]



Geometric Routing without Geometry

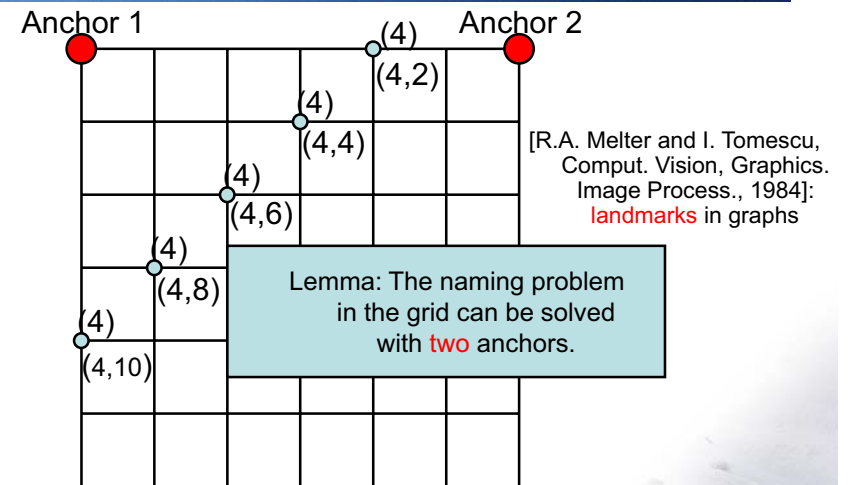
- For many applications, like routing, finding a **realization** of a UDG is **not mandatory**
- Virtual coordinates merely as **infrastructure** for geometric routing
 - **Pseudo geometric** coordinates:
 - Select some nodes as **anchors**: a_1, a_2, \dots, a_k
 - Coordinate of each node u is its **hop-distance** to all anchors: $(d(u, a_1), d(u, a_2), \dots, d(u, a_k))$



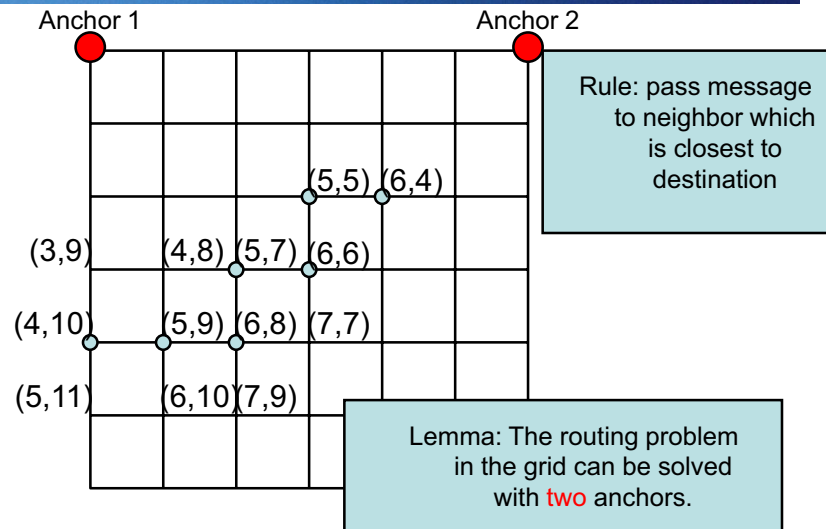
- Requirements:
 - each node uniquely identified: **Naming Problem**
 - routing based on (pseudo geometric) coordinates possible: **Routing Problem**



Pseudo-geometric routing in the grid: Naming

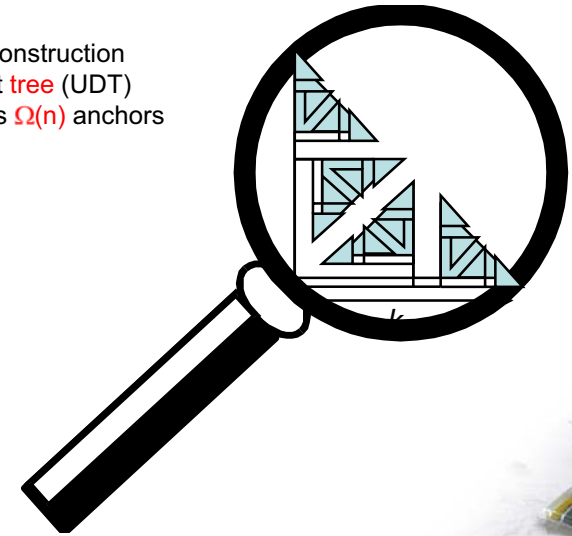


Pseudo-geometric routing in the grid: Routing



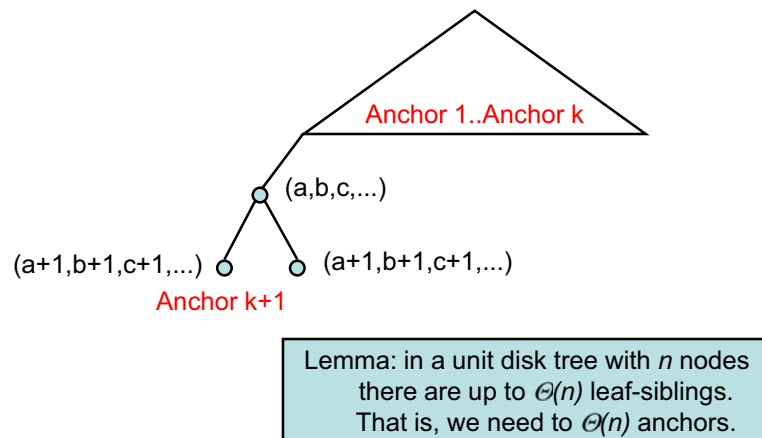
Problem: UDG is usually not a grid

- Recursive construction of a unit dist tree (UDT) which needs $\Omega(n)$ anchors



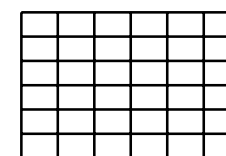
Pseudo-geometric routing in the UDT: Naming

- Leaf-siblings can only be distinguished if one of them is an anchor:

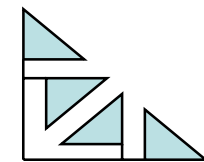
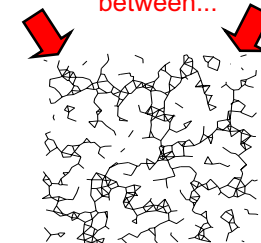


Pseudo-geometric routing in the ad hoc networks

- Naming and routing in grid quite good, in previous UDT example very bad
- Real-world ad hoc networks are very probable neither perfect grids nor naughty unit disk trees



Truth is somewhere in between...



Summary of Results

- If position information is available geo-routing is a feasible option.
- **Face routing** guarantees to deliver the message.
- By restricting the search area the efficiency is **OPT²**.
- Because of a lower bound this is asymptotically optimal.
- Combining greedy and face gives efficient algorithm.
- **3D** geo-routing is **impossible**.
- Even if there is no position information, some ideas might be helpful.

- Geo-routing is probably the only class of routing that is well understood.
- There are **many adjacent areas**: topology control, location services, routing in general, etc.



Open problem

- One of the most-understood topics. In that sense it is hard to come up with a decent open problem. Let's try something wishy-washy.

- For a 2D UDG the efficiency of geo-routing can be quadratic to an optimal algorithm (with routing tables). However, the worst-case example is quite special. Open problem: How much information does one need to store in the network to guarantee only constant overhead?
 - Variant: Instead of UDG some more realistic model
 - How can one maintain this information if the network is dynamic?

