From Shared Memory to Message Passing

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Some parts of the lecture, parts of the „Skript“ and exercises will be based on the lectures of

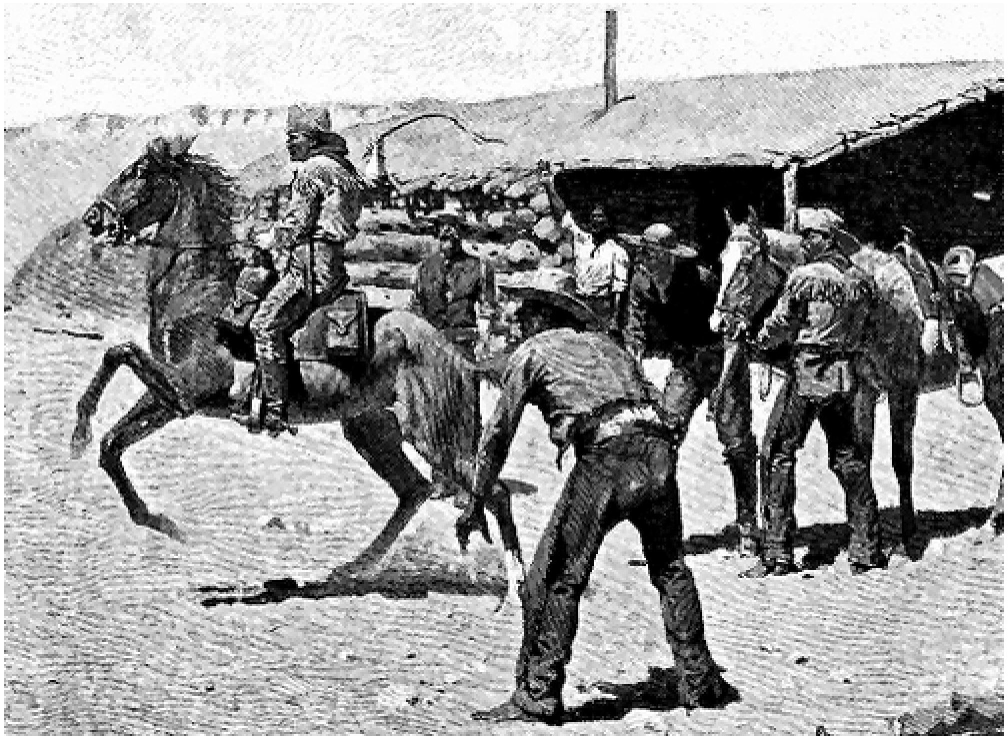
Prof. Roger Wattenhofer at ETH Zurich

and

Prof. Christian Scheideler at University Paderborn.

Message Passing

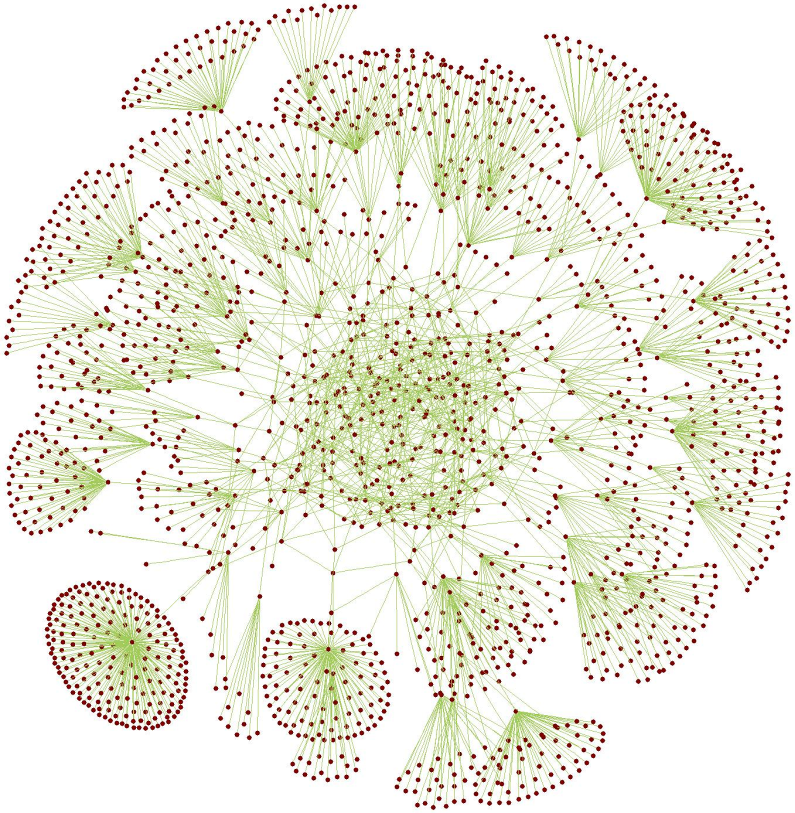
Many distributed systems do not have a shared memory, but pass messages along networks: social networks...



Pony express: mail along road networks

Message Passing

Many distributed systems do not have a shared memory, but pass messages along networks: ... Internet networks ...



Internet

Message Passing

Many distributed systems do not have a shared memory,
but pass messages along networks: ... but also wireless networks.

Multi-hop sensor networks

Fundamental Questions: Communication Network?

Sometimes topology can be chosen (e.g., peer-to-peer networks)!

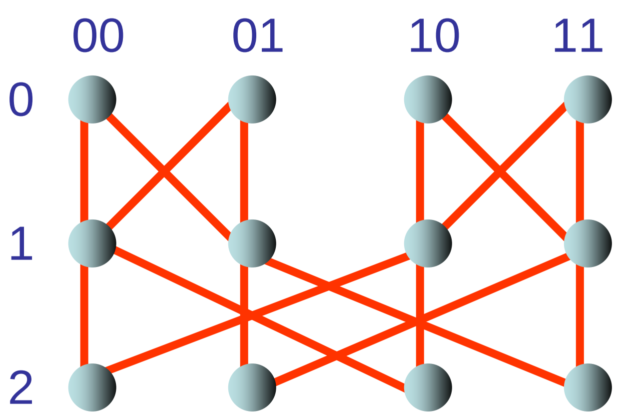
What communication networks / architectures are „good“?

A comparison.

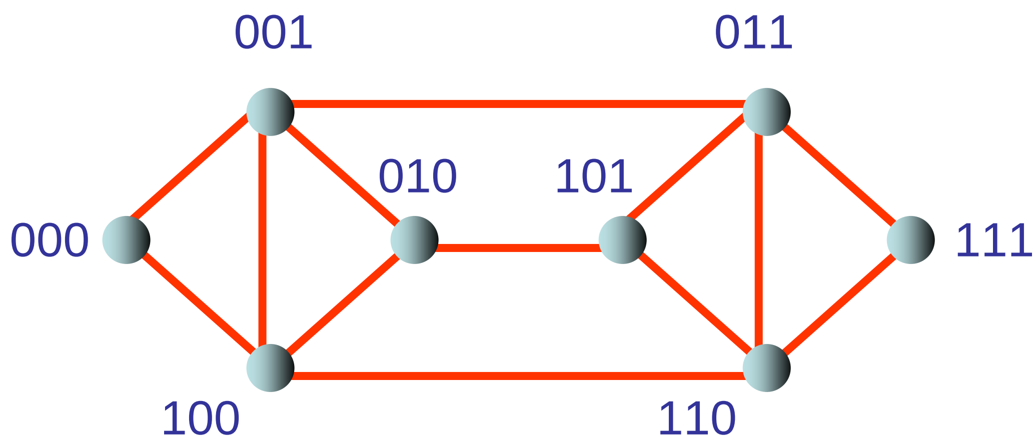
(degree-diameter tradeoff, network expansion, routing, robustness under dynamics, ...)

Line: simple, but long communication delays and not robust?

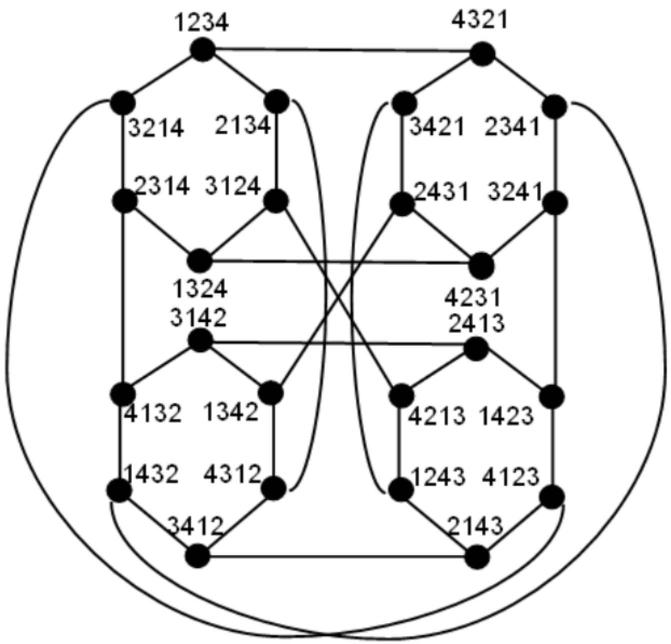
Butterflies?



De Bruijn graphs?



What is the „best“ degree / diameter tradeoff?



Line: diameter n-1, degree 2

Clique: diameter 1, degree n-1

Hypercube: diameter log n, degree log n

Can we reduce both?

Yes.

It must hold that $\textbf{degree}^{\textbf{diameter}}$ **> n** (why?)

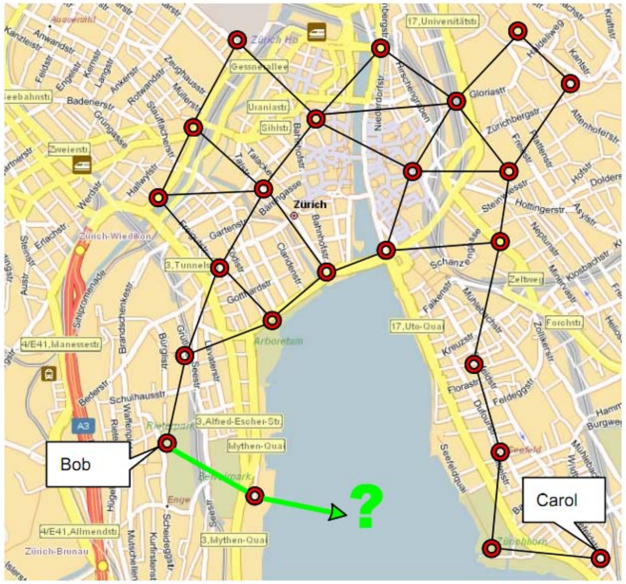
Pancake graphs: log n / loglog n diameter, log n / loglog n degree

How to make these topologies robust and dynamic? (E.g., continuous-discrete approach)

Fundamental Questions: Algorithms

Fundamental tasks on networks:

- Routing: sending a message from node A to node B?
- Broadcasting: sending a message to all nodes?
- Aggregation: Finding the most frequent element in the network? The node measuring the hottest temperature? Etc.
- Electing a leader
- Coloring the network (e.g., frequency spectrum allocation in wireless networks), computing independent sets, etc.



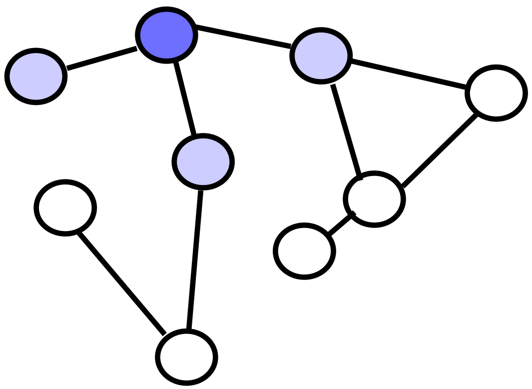
Complexity evaluation:

- Distributed runtime: number of communication rounds until task fulfilled?
- Message complexity: number (and size) of messages to be transmitted?
- Local complexity of algorithm: Complexity of algorithm in node?

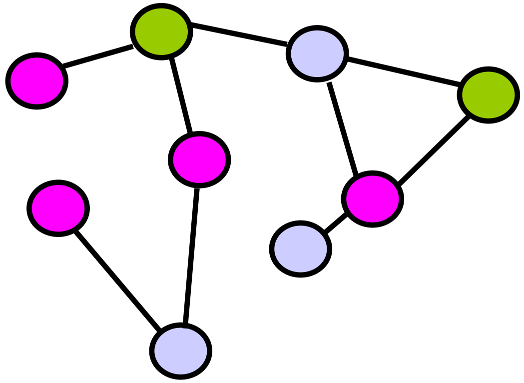
Example: Local Vertex Coloring

Local Algorithm

Each processor / node must act based on information about its k-hop neighborhood! (Fast and efficient algorithms, good under dynamics!)



Sometimes local algorithm can approximate even NP-hard problems quite well and fast!



Vertex Coloring

Nodes should color themselves such that no adjacent nodes have same color – but minimize # colors!

Used, e.g., for wireless spectrum allocation...

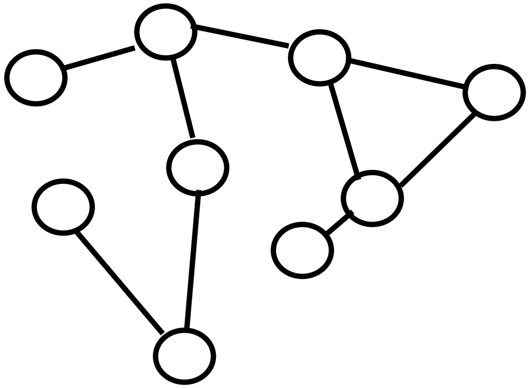
Local Algorithm

Simplified round model, all nodes execute the same protocol, ...

In one round:

1. send messages (message complexity)
2. receive messages
3. process messages (local computation complexity)

Number of rounds until termination: time complexity



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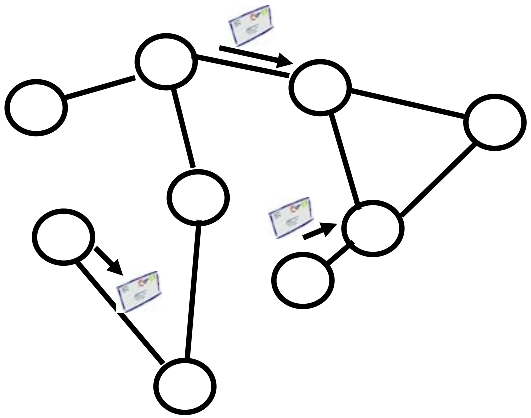

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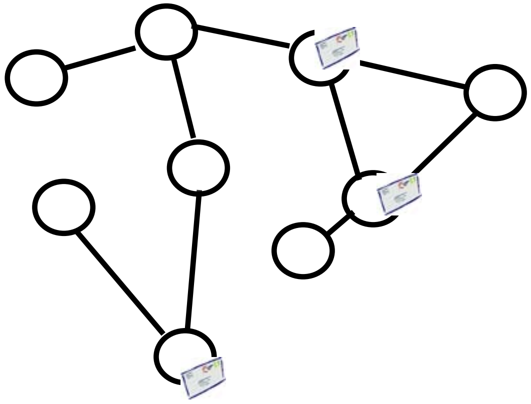

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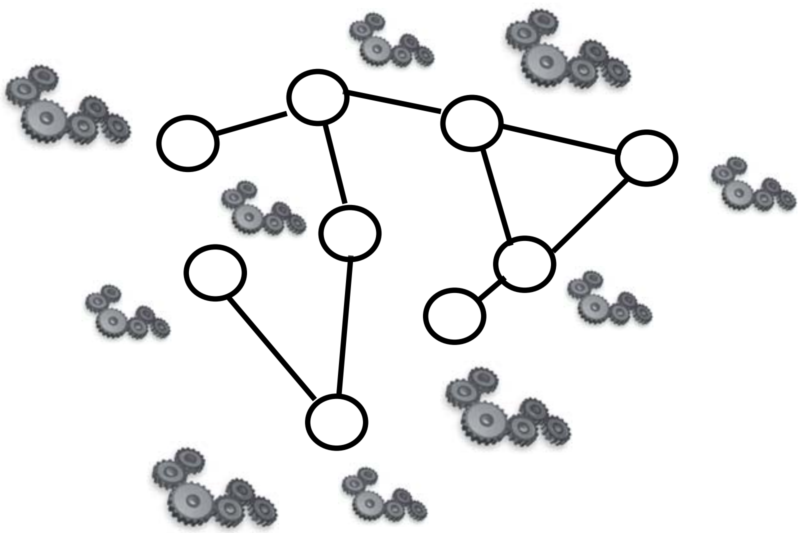
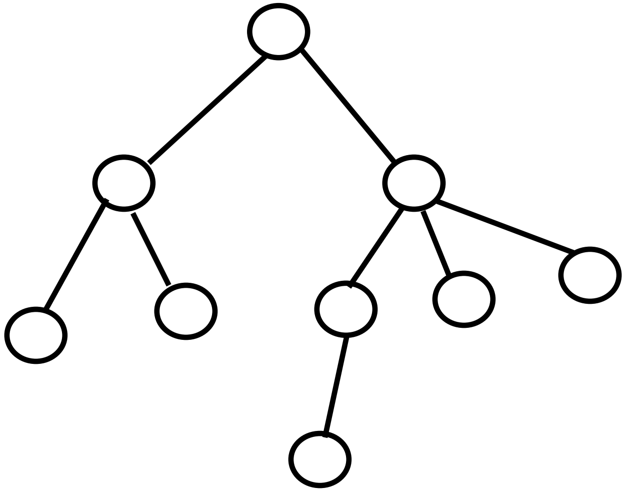


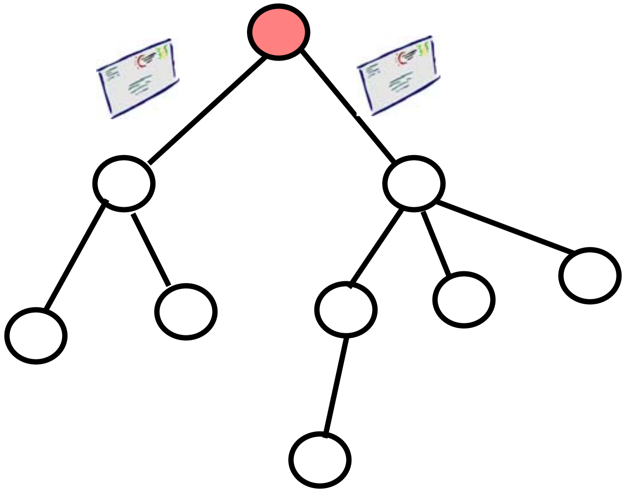
Local Vertex Coloring for Rooted Tree?

Ideas? (Assume, e.g., arbitrary but unique IDs are given at nodes...)



Local Vertex Coloring for Tree?

Two colors suffice: root sends binary message down...



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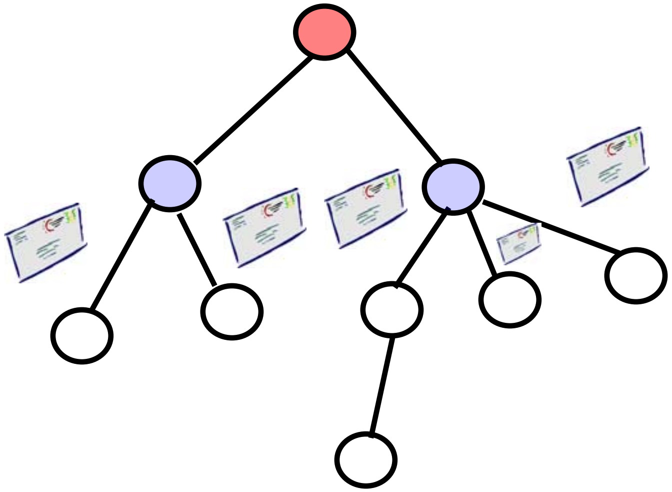

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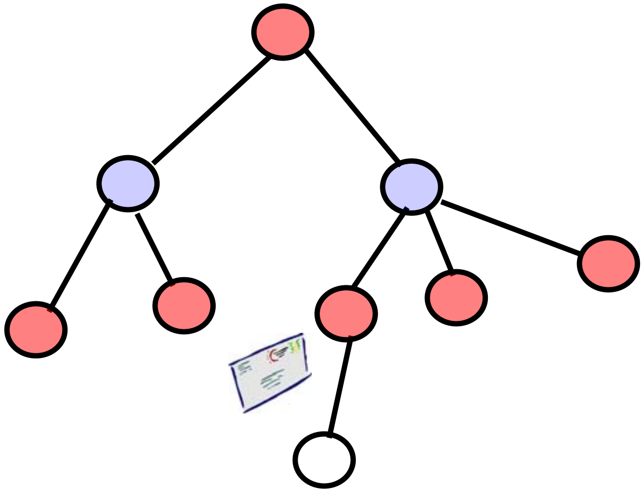

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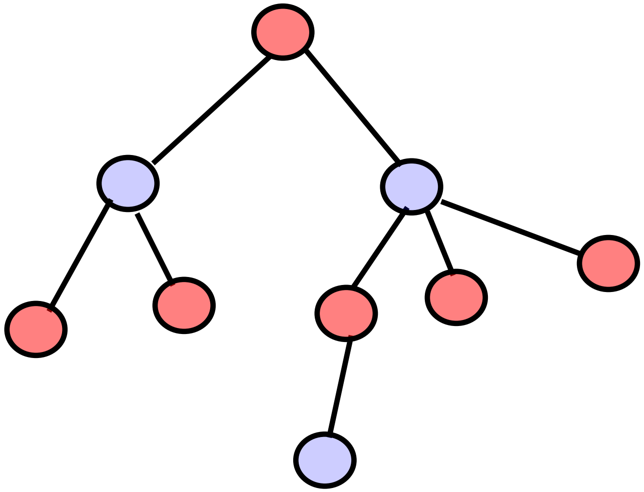


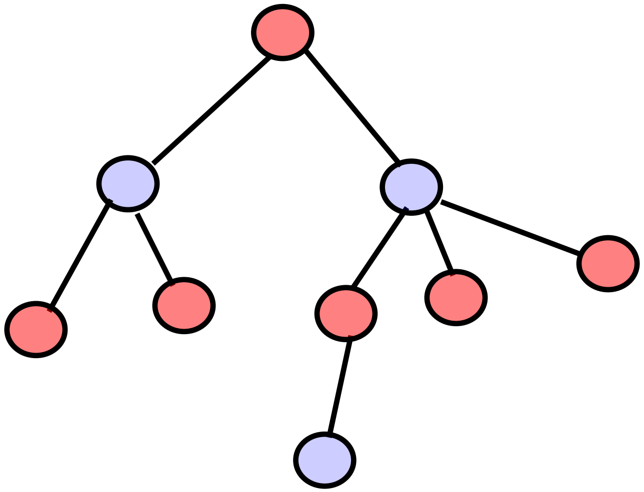
Time complexity?

Message complexity?

Local compuations?

Local Vertex Coloring for Tree?

Can we do faster than diameter of tree?!



Yes! With **constant** number of colors in

log*(n) time!!

One of the fastest non-constant time algos that exist! (...
Besides inverse Ackermann function or so)

(log = divide by two, loglog = ?, log* = ?)

log* (# atoms in universe) ≈ 5

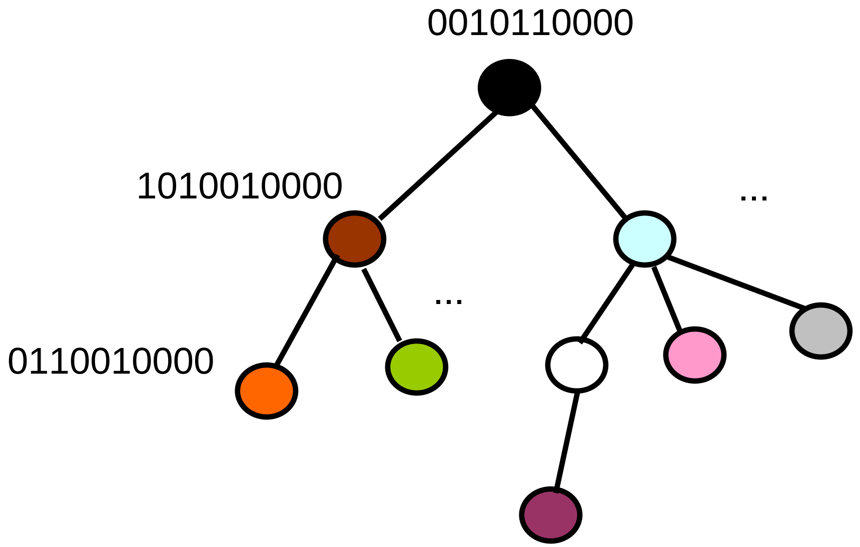
Why is this good? If something happens (dynamic network),
back to good state in a sec!

How? You will learn. ☺

There is a lower bound of log-star too, so that‘s optimal!

How does it work?

Initially: each node has unique log(n)-bit ID = legal coloring
(interpret ID as color => n colors)



Idea:

root should have label 0 (fixed)

in each step: send ID to c_v to all children;

receive c_p from parent and interpret as little-endian bit string: c_p=c(k)...c(0)

let i be smallest index where c_v and c_p differ

set new c_v = i (as bit string) || c_v(i)

until $c_v \in \{0,1,2,...,5\}$ (at most 6 colors)

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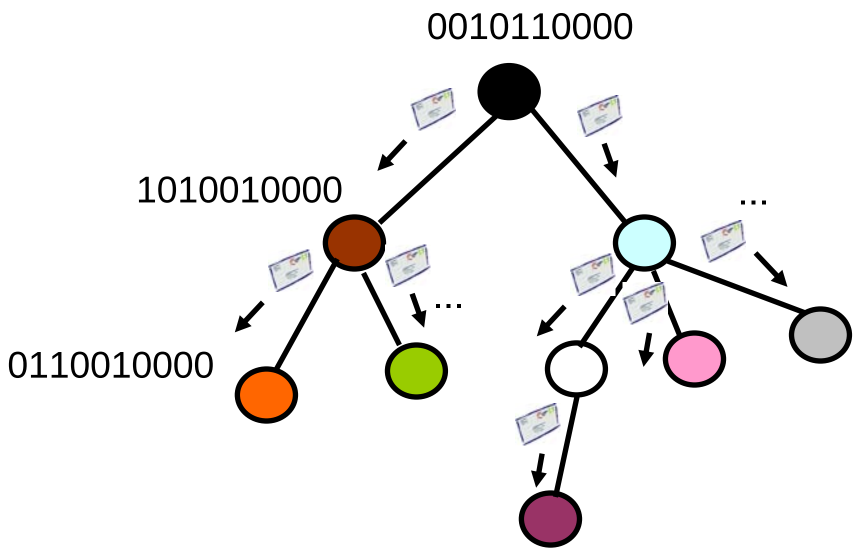

Round 1

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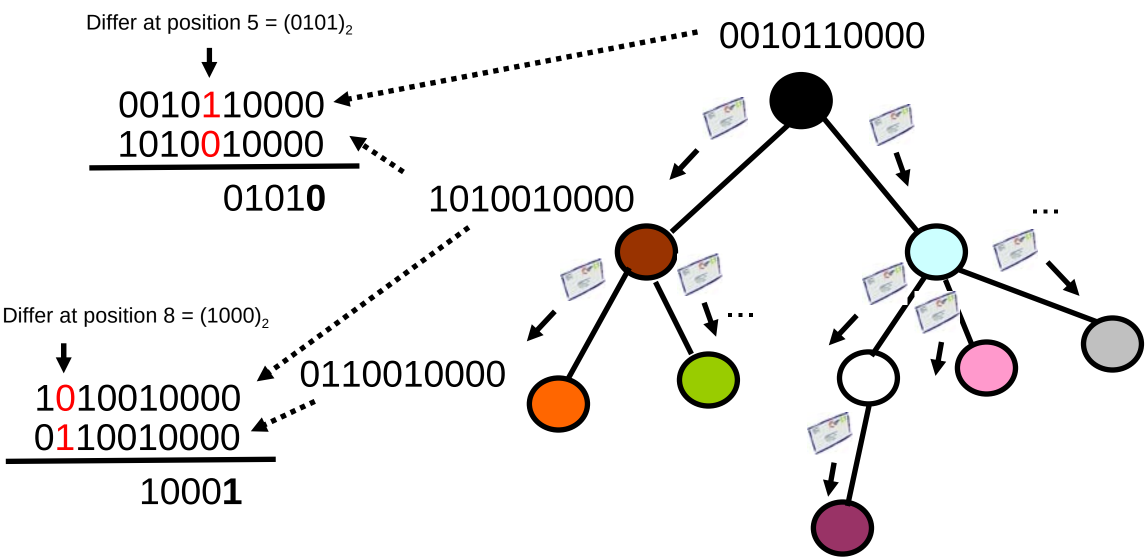

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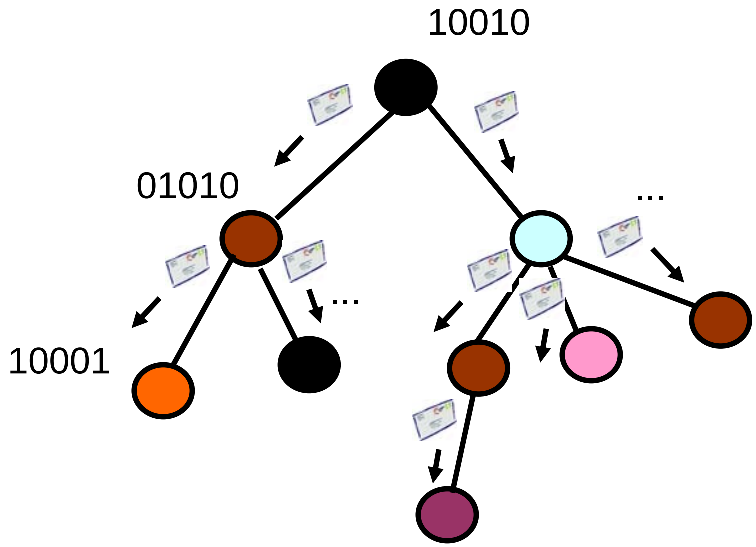

Round 2

Idea:

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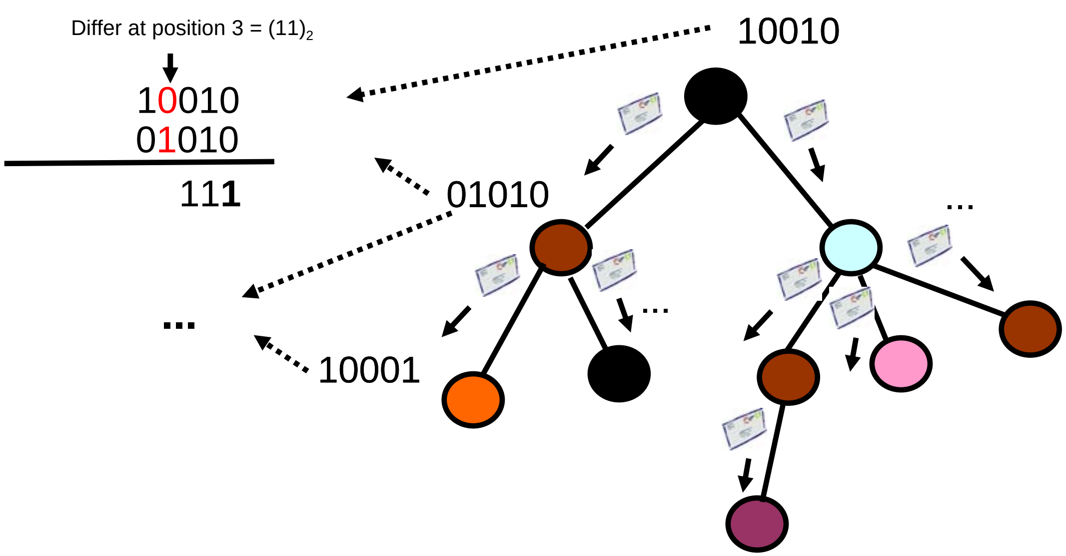

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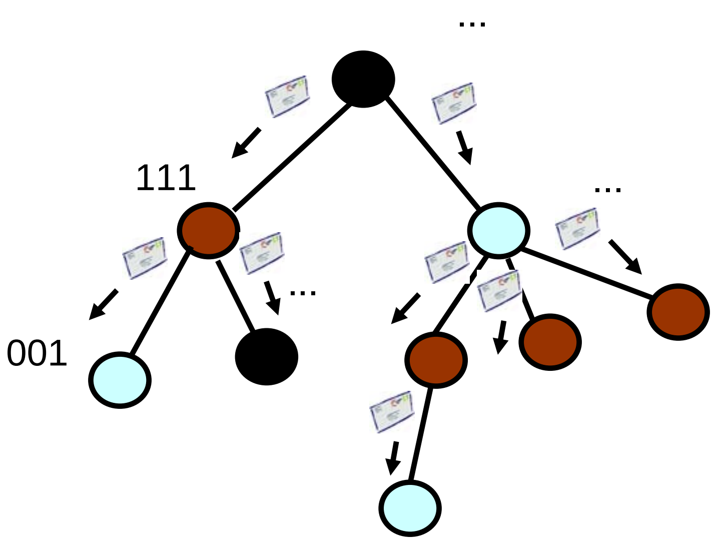

Round 3, etc.

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Why does it work?

Why is this log* time?!

Idea: In each round, the size of the ID (and hence the number of colors) is reduced by a log factor:
To index the bit where two labels of size n bits differ, log(n) bits are needed!
Plus the one bit that is appended...

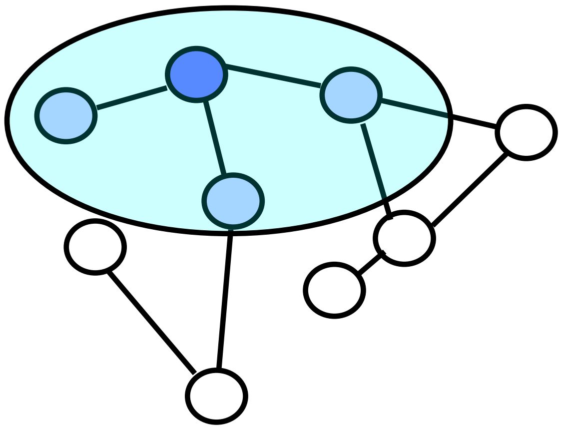
Why is this a valid vertex coloring?!

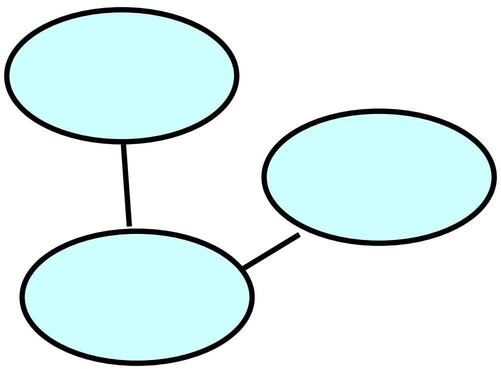
Idea: During the entire execution, adjacent nodes always have different colors (invariant) because:
IDs always differ as new label is index of difference to parent plus own bit there (if parent would differ at same location as grand parent, at least the last bit would be different).

Lower Bound

r-hop View

R-hop view of a node v defined as set of states of all nodes in r-hop neighborhood of v.





Neighborhood Graph

Vertices of neighborhood graph are r-hop views; vertices are connected if views could result from adjacent nodes!

Observe: Any deterministic vertex coloring algorithm can be seen as mapping r-hop neighborhoods to colors.

Observe: Chromatic number of neighborhood graph (classic coloring) implies possible local algorithm coloring.

=> Gives us lower bound of possible colorings with r-neighborhood!