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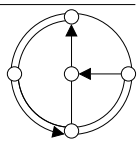
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich



FS 2008 Prof. R. Wattenhofer / Dr. F. Kuhn / T. Locher / Y. A. Oswald / C. Lenzen

Principles of Distributed Computing
Exercise 1

1 Vertex Coloring

In the lecture, a simple distributed algorithm which colors an arbitrary graph with $\Delta + 1$ colors in n synchronous rounds was presented (Δ denotes the largest degree, n the number of nodes of the graph).

a) What is the message complexity, i.e., the total number of messages the algorithm sends in the worst case?

b) Does the algorithm also work in an asynchronous environment? If yes, formulate the asynchronous equivalent to the algorithm, if no, describe why.

2 Coloring Trees

We learnt in the lecture that any (directed) tree can be colored with 3 colors in $\log^* n + O(1)$ time. In the first phase of the algorithm, the nodes acquire a color in the range $\mathcal{R} := \{0, \ldots, 5\}$.

The problem with this algorithm is that nodes do not know when the first phase is over: A node v cannot simply decide to be done once its color is in $\mathcal{R}$ as long as its parent w might still change its color. Even if the color of w is also in $\mathcal{R}$, w might receive a message from its parent, forcing w to change its color (potentially to node v's color!).

Since this is a difficult problem, we will try to find a solution for the ring topology first. Formally, the ring is the graph $G = (V, E)$, where $V = \{v_1, \ldots, v_n\}$ and $E = \{\{v_i, v_j\} \mid j = i + 1 \,(\mathrm{mod}\, n)\}$. You can assume that G is a *directed* ring, i.e., nodes can distinguish between their left and right neighbor.

a) Show how the log-star coloring algorithm for trees can be adapted for the ring!

b) Once the color of node v in the ring is in $\mathcal{R}$, it wants to reduce the numbers of colors used to 3. Show how this reduction phase works even if the first phase may not have terminated in some other parts of the ring![1]
Hint: You can use additional colors to segment the ring and to solve the problem locally!

c*) Propose a solution to the color reduction problem that works on any directed tree![2]

[1] Note that a node cannot wait until it knows that all other nodes are ready to start the second phase as this would require time in the order of n!

[2] Problems marked with an asterisk (*) are hard. Example solutions to these problems will not be provided. However, anybody who solves such a problem will receive a price!